Models to Predict Mechanical Responses in Rigid Pavements

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Abstract: The most recent tendencies in pavement engineering design are directed to the return of the use of fundamental mechanical principles of engineering to predict design life through the concept of damage. Damage is associated with the relation between (1) the number of repetitions a material can resist until failure and (2) the predicted repetitions the designer is expecting during a period of time. Generally, mechanical responses as strains, stresses, and displacements are used to calculate the number of repetitions until a specific failure. In rigid pavements, there are analytical solutions that range from simple Westergaard's closed-form formulas to complex numerical solutions (as discrete-element methods and finite-element methods). This paper describes work done to develop an additional option in the middle: models calibrated to have the simplicity of the closed-form formulas and the accuracy of the finite-element methodology. Those models were then included in a graphical user interface, which will be used as the structural response engine in local mechanistic-empirical (M-E) design software. **DOI: 10.1061/JTEPBS.0000027.** *© 2017 American Society of Civil Engineers.*

Introduction

Current pavement design practices are evolving toward mechanistic-empirical (M-E) design philosophies. Mechanistic-empirical methodologies use performance models to predict distress development from the critical mechanical responses of the pavement structure. Those mechanical responses are a function of the material proprieties, climatic conditions, and transit distribution.

The traditional approach to solve this problem is based on closed-form solutions, which are easy to use but are based in rigid theoretical assumptions, limited mechanical knowledge, and harsh simplifications. These equations are not suitable for design purposes because of the precision and robustness needed. To introduce more-realistic solutions, numeral approximated solutions have been developed on the basis of the finite-element method. Those solutions have been proved accurate and practical through the years. However, there are few available specific programs using this methodology to analyze rigid pavements, and the cost associated to acquire license and training is high. Even if those cost can be covered by design agencies, time-related constrains on the modeling of thousands of material proprieties, loads, and environmental conditions make the use of finite element unpractical on M-E design guides.

Considering those aspects and the current advance in the calibration and the definition of the Costa Rican mechanistic-empirical pavement design guide (CR-ME), it was necessary to develop a simple, accessible, fast, cost-effective, and accurate methodology to predict rigid pavement responses. The work consisted in analyzing 19,683 structures, defined from a parametric combination of typical mechanical and geometrical properties in rigid pavements in Costa Rica, by using the software *ISLAB2000* (a finite-element-based program) (Khazanovich et al. 2000). From the generated database, statistical and computational models were adjusted to predict mechanical responses on rigid pavements. The models were based on the multiple linear regression (MLR) and artificial neural network (ANN) methodology.

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Methodology

The project can be described simply as a three-stage process. The first stage included the definition of an extensive structures database, which was then analyzed with the finite-element software *ISLAB2000* in the second phase. Finally, from this analysis, results were extracted and then used in the calibration of the statistical models.

The structure database definition involved an extensive bibliographical revision of multiple subjects, including analysis and design of rigid pavements, local material characterization reports, local climate and environmental elements, temperature distributions, and regional construction techniques. All of these aimed toward the construction of a comprehensive database, comprising most of the broad range of possibilities regional designers have for rigid pavement design. Special concern was placed on local material proprieties and their particularities as they play an important role in the structural analysis. This revision defined eight variable parameters, which are most related to variations on the structural responses.

Each parameter was assigned with three different discrete values according to local variation and common design practice. The ranges were defined to consider most of their normal variation on local conditions, minimizing the future need of models to extrapolate scenarios. These parameters and their discrete values are shown in Table 1. All possible combinations of these values and corresponding critical load positions defined a total of 19,683 different structures that were analyzed in the finite-element analysis with *ISLAB2000*. Other parameters related with more stable or

Table 1. Variables and Values Used in the Rigid Pavement Structures Modeling

Variable	Value
Joint spacing [m (ft)]	3.7 (12)
	4.6 (15)
	5.2 (17)
Concrete elastic modulus [GPa (ksi)]	27.6 (4,000)
	34.5 (5,000)
	41.2 (6,000)
Slab thickness [cm (in.)]	15.2 (6)
	33.0 (13)
	43.2 (17)
Temperature differential [°C (°F)]	−10 (−18)
	−2.3 (−4)
	6.6 (12)
Subgrade reaction modulus [MPa/m (psi/in.)]	27.1 (100)
	54.3 (200)
	81.4 (300)
Base elastic modulus [GPa (ksi)]	0.34 (50)
	1.90 (250)
	3.45 (500)
Dowel diameter [cm (in.)]	0 (0)
	2.5 (1)
	3.8 (1.5)
Load transfer efficiency (%)	10
	50
	80

Table 2. Fixed Parameters Used in the Rigid Pavement Structures Modeling

Parameters	Value
Base thickness [cm (in.)]	25 (10)
Concrete Poisson coefficient	0.175
Granular base Poisson coefficient	0.35
Concrete thermal expansion coefficient [1/°C (1/°F)]	9.9×10^{-6} (5.5×10^{-6})
Concrete density [kg/m^3(lb/in.^{3})]	2,408 (0.0870)
Wheel wander [m (in.)]	0.3 (12)
Tire pressure [kPa (psi)]	(110)
Tire aspect ratio	0.5
Slab width [m (ft)]	3.65 (12)
Shoulder width [m (ft)]	1.80 (6)

noncritical aspects of pavement design were assumed constant on all models; those parameters are shown in Table 2.

Geometrical dimensions used in the model were defined with a three-slab wide configuration, including a 1.8-m (12-ft) shoulder and an unloaded traffic slab. The system length was varied according to load positions, minimizing the model size while always maintaining realistic boundary conditions with unloaded slabs on both system ends. Finite-element mesh size was defined after repeated test on model convergence; finally, it was determined that a 7.5-cm (3-in.) finite-element size was sufficient to guarantee model convergence on the most demanding scenarios.

Critical responses location were based in the type of distresses [bottom-up and top-down transverse cracking, transverse joint faulting, and international roughness index (IRI) (ERES Consultants and ARA 2004)] included in the Mechanistic-Empirical Pavement Design Guide (MEPDG) by AASHTO for jointed plain concrete pavements (JPCP) (ERES Consultants and ARA 2003): (1) longitudinal tensile bending stress on the top of the slab; (2) longitudinal tensile bending stress at the bottom of the slab; and (3) differential vertical deflections across transverse joints.

The load applied corresponds to the vehicle T3-S2 [Type 9, five-axle, single-trailer truck in the Federal Highway Administration (FHWA) classification], predominantly used in Costa Rica. The load was determined as 6,000 kg (13,200 lb) in the frontal axle and 16,500 kg (35,200 lb) in the traction and trailer axle, considering the maximum load allowed in Costa Rica to the T3-S2 (Ministerio de Obras Públicas y Transportes 2003).

As a validation method, the authors defined a verification database, comprising a smaller number of structures not included in the calibration phase, which also included parameters outside of the calibration database ranges. The main goal with this database was to observe model behavior outside of the calibration ranges. Special concern was taken with slab thickness of less than 15 cm (6 in.), as it is one of the main parameters in the structural analysis (Huang 2004).

The third and last phase of the project included the calibration of multiple statistical models that correlate different rigid pavement structure parameters with the mechanical responses obtained with finite-element analysis. The main goal was to calibrate linear regression models as a way of defining a simple set of equations with an adequate precision for M-E design purposes.

As an alternative solution, artificial neural network models were trained with a multiple back-propagation algorithm included in the open-source (released under the general public license GPLv3) software *Multiple Back-Propagation* (Lopes and Ribeiro 2003). Artificial neural network models have been proven as a reliable tool for solving similar problems, specially related to modulus back-calculation (Birkan 2006), rigid pavement airfield pavements analysis (Ceylan et al. 1999), and structural analysis engine of the MEPDG (ERES Consultants and ARA 2003). This methodology has the potential to calibrate a better model, sacrificing some ease of use, as ANN models require the use of a computational algorithm; therefore, it would be necessary to develop a customized application to evaluate and distribute the results. This issue was resolved with the development of *ApRIGID 1.0* software, which simplifies the use of all the calibrated models in the design process of rigid pavements.

The project originated eight different models, four multiple linear regression models, and four based in the artificial neural network methodology. All models were statistically validated; tests applied to each model included coefficient significance, residual normality, and residual homoscedasticity. The models were also evaluated on certain scenarios in which they are forced to extrapolate data out of their calibration ranges as a way to analyze model behavior on those extreme but plausible cases.

Results and Discussion

MLR-Based Models

Linear regression models were calibrated by using the stepwise technique to find the best predictors for each model; several different combinations of variables were evaluated. As a starting point, Westergaard closed-form solutions were used; each initial regression predictor tried to replicate the form of the known equation. Modifications and new variable transformations were made as each test model was calibrated and analyzed. Finally, a statistical variable reduction approach was used to reduce the number of total predictors. In all cases, it was found that eight different variables were sufficient to explain the data variability and keep a compact model that is easy to use according to the project objectives.

Four different models were calibrated: longitudinal tensile stress at the top of the slab caused by center load case (Model A) [Eq. (1)]; longitudinal tensile stress at the bottom of the slab caused by edge loading (Model B) [Eq. (2)]; differential deflections between adjacent slabs caused by corner loading on nondoweled pavements (Model C1) [Eq. (3)]; and differential deflections between adjacent slabs caused by corner loading on doweled pavements (Model C2) [Eq. (4)]

Model A

$$\sigma_{y,s} = -281.12 + 102.9 \times L - 63.64 \times \Delta t - 3.0 \times LTE - \frac{5.083 \times E \times L \times \Delta t}{l} + \frac{1}{h^2}\left(-60{,}509.9 \times L + 52{,}245.4 \times l - \frac{116.37 \times E \times l \times \Delta t}{L}\right) + \frac{24.061 \times l \times \Delta t}{L} \quad (1)$$

Model B

$$\sigma_{y,b} = 29.942 + 4.046 \times \Delta t - 0.338 \times LTE + \frac{0.911 \times LE\Delta t}{1{,}000 \times l} - \frac{3.779 \times l\Delta t}{L} + \frac{1}{h^2}\left(\frac{21.094 \times \Delta t \times E}{1{,}000} - 4.478 \times E_b - 63.766 \times L + 1{,}053.909 \times l\right) \quad (2)$$

Model C1

$$\delta_{1-2} = -0.0223 - \frac{0.1599 \times L}{1{,}000} - \frac{1.4885 \times E}{10^6} - \frac{1.4528 \times h}{1{,}000} - \frac{0.1230 \times LTE}{1{,}000} + \frac{2.967 \times l}{1{,}000} + \frac{1}{k \times l}\left(\frac{270.62 \times k}{1{,}000} - \frac{8{,}038.78}{l^2}\right) + \frac{1{,}215.96}{\sqrt{E \times h^3 \times k}} \quad (3)$$

Model C2

$$\delta_{1-2} = 5.584 \times 10^{-3} - \frac{4.507 \times L}{10^6} - \frac{1.487 \times k}{10^6} - \frac{2.494 \times Ø}{1{,}000} - \frac{18.944 \times LTE}{10^6} + \frac{36.059 \times l}{10^6} + \frac{3.982 \times 1}{l \times 1{,}000} + \frac{7.763 \times Ø \times LTE}{10^6} + \frac{34.139 \times l^4}{E \times h^3 \times 1{,}000} \quad (4)$$

where $\sigma_{y,s}$ = longitudinal tensile stress at the top of the slab (kPa); $\sigma_{y,b}$ = longitudinal tensile stress on slab bottom (MPa); δ_{1-2} = differential deflections between slabs (cm); L = slab length (m); Δt = temperature differential through slab thickness (°C); LTE = load transfer efficiency (%); E = concrete elastic modulus (GPa); E_b = granular base elastic modulus (GPa); h = slab thickness (cm); k = coefficient of subgrade reaction (MPa/m), Ø = dowel diameter (cm); and l = radius of relative stiffness [Eq. (5)]

$$l = \left(\frac{E \times h^3}{k}\right)^{0.25} \quad (5)$$

Every model has values of adjusted R^2 of more than 0.95. Reviewing the null hypothesis of homoscedasticity to justify the use of ordinary least squares (OLS) through White test, there was proved heteroscedasticity in residuals; consequently, it was necessary to use generalized least squares (GLS) to correct the variance. The general statistical results obtained are show in Table 3, in which the significance of every variable is more than 95% of confidence ($t_{\mathrm{crit}} = 2.576$).

ANN-Based Models

The same data set was used to calibrate an ANN with a feed-forward network; topology was defined as 8-15-1 (eight input neurons, 15 neurons in a hidden layer, and a single output). This was done with the multiple back-propagation algorithm based on the implementation of the open-source software *Multiple Back-Propagation*. Convergence criteria were defined at a root-mean-square error of 0.001. A small script was then constructed to calculate residual errors and proceed to a direct performance comparison between both ANN and MLR models.

Comparison between MLR and ANN Models

In most cases, the artificial neural network model results were substantially better than those of the multiple linear regression model. A numerical comparison between both methodologies can be found in Table 3.

In Models A and B, the artificial neural network model results were substantially better than those of the multiple linear regression model. In the first case, the mean residual errors of the MLR model were computed at 9.8%, whereas ANN errors on the same data set were only 5.6%. Figs. 1(a and b) compare both models, and it is evident that the ANN has a better fit on all stress ranges. In the second case, Model B, mean residual errors in the regression model were computed at 11.0%, whereas ANN errors on the same data set were only 6.60%. Figs. 2(a and b) compare both models, and it is evident that the ANN has a better fit on all stress ranges.

Meanwhile, performance differences between MLR and ANN on Models C1 and C2 are virtually nonexistent as shown on Figs. 3 (a and b) and 4(a and b), respectively. Mean errors were computed at approximately 7.5 and 3.2%, respectively. Multiple linear regression showed a slight advantage in these cases.

During model verification, it was observed that the model was unable to predict in an acceptable way stresses of less than 210 kPa

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Table 3. Statistical Analysis of Multiple Linear Regression and Artificial Neural Networks

Model	MLR						ANN residual error (%)
	R^2, adjusted	Variable	Coefficient	Standard error	t	Residual error (%)	
A	0.987	$V1$	4.551	0.082	55.54	9.8	5.6
		$V2$	−5.128	0.128	−40.08		
		$V3$	−0.435	0.006	−72.74		
		$V4$	−1.072	0	−82.52		
		$V5$	−414.61	4.936	−83.99		
		$V6$	2943.414	10.482	90		
		$V7$	5.11	0.076	67.01		
		$V8$	−26.41	0.001	−31.53		
		Constant	−40.772	1.505	−27.09		
B	0.987	$V1$	4.046	0.148	27.32	11.0	6.6
		$V2$	−0.338	0.006	−56.85		
		$V3$	0.001	0	61.4		
		$V4$	0.021	0.001	34.99		
		$V5$	−4.478	0.056	−80.19		
		$V6$	−63.766	3.438	−18.55		
		$V7$	1,053.909	7.667	137.46		
		$V8$	−3.779	0.094	−40.41		
		Constant	29.942	0.529	56.65		
C1	0.974	$V1$	−0.16	0.018	−9.03	6.8	8.3
		$V2$	−1.489	0.068	−22.05		
		$V3$	−1.453	0.071	−20.46		
		$V4$	−0.123	0.002	−72.94		
		$V5$	2.967	0.043	69.17		
		$V6$	1,215.96	29.987	40.55		
		$V7$	0.271	0.02	13.57		
		$V8$	−8,038.778	285.264	−28.18		
		Constant	−0.022	0.003	−7.83		
C2	0.968	$V1$	−4.507	0.7	−6.44	3.12	3.3
		$V2$	−1.487	0.07	−21.22		
		$V3$	−2.494	0.015	−162.27		
		$V4$	−18.944	0.372	−50.87		
		$V5$	36.059	1.606	22.46		
		$V6$	3.982	0.272	14.63		
		$V7$	7.763	0.28	27.76		
		$V8$	34.139	1.974	17.3		
		Constant	0.006	0	113.63		

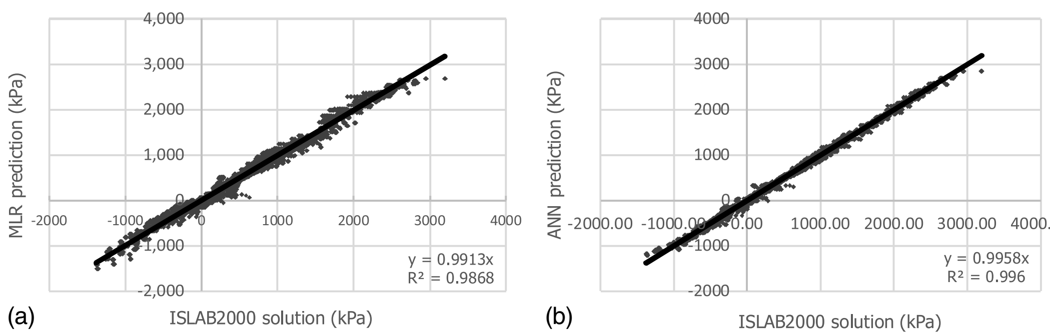

Fig. 1. Model A verification results: (a) MLR; (b) ANN

(30 psi). Scenarios in which critical tensile stresses are extremely low or even in compression are not predicted well by the model. This behavior was also observed in Model B. Local calibrated fatigue models (Monge 2013) showed that stresses less than 210 kPa (30 psi) do not cause a quantifiable fatigue damage on concrete. A regular 4.5-MPa modulus of rupture concrete has, according to those models, a capacity of more than 10^{33} allowed load applications before failure in those stress ranges, which in practical terms is almost an infinite quantity for pavement design.

Model behavior with structures outside of their calibration ranges showed adequate predictions on most cases. The exception was found on slabs thickness less than 10 cm (4 in.). In those cases, all responses were underestimated by the model. This problem was observed in all scenarios and is related with extreme structural conditions on thin slabs. Slab thickness of less than 10 cm (4 in.) are not commonly found on rigid pavement structures, as they have a poor fatigue performance. Thin slab design generally uses specific design guidelines independent of concrete pavement design guides.

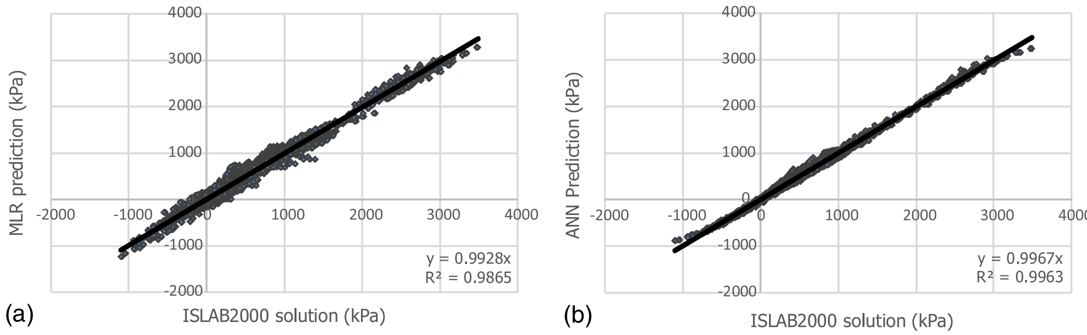

Fig. 2. Model B verification results: (a) MLR; (b) ANN

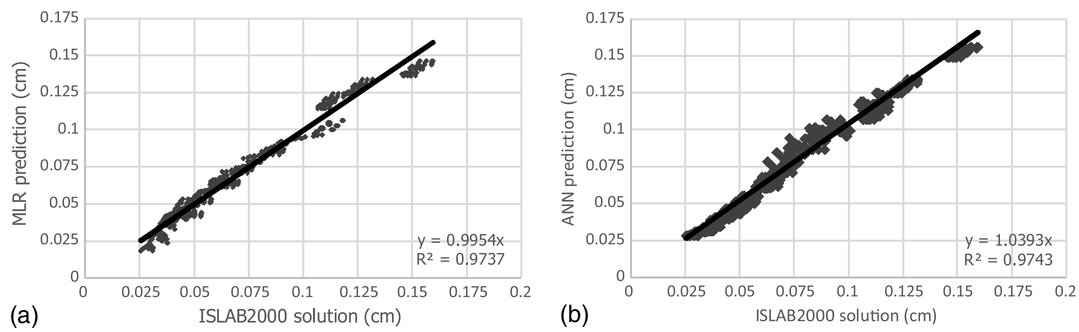

Fig. 3. Model C1 verification results: (a) MLR; (b) ANN

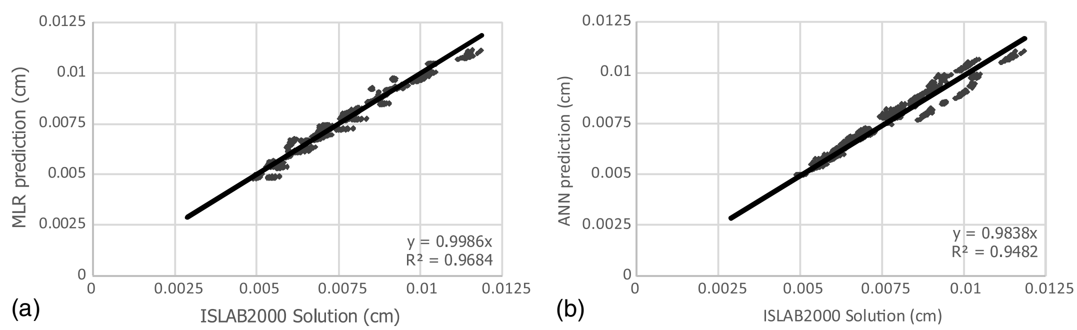

Fig. 4. Model C2 verification results: (a) MLR; (b) ANN

This model inability to predict thin slab stresses accurately is considered as a noncritical limitation that does not jeopardize model use on regular rigid pavement design.

ApRIGID 1.0

A graphical user interface (GUI) called *ApRIGID* was developed to facilitate the use of both the MLR and ANN models by potential users. This software provides a fast and simple option to obtain critical responses necessary in the design of rigid pavements with M-E philosophies. It was developed on the programming language *Java* and requires at least the *Java Runtime Environment 1.6.0*.

As shown in Fig. 5, *ApRIGID 1.0* allows to define eight parameters of a rigid pavement structure: joint spacing, concrete elastic modulus, slab thickness, temperature differential, subgrade reaction coefficient, granular base elastic modulus, dowel diameter, and longitudinal joint load transfer efficiency. The results obtained are longitudinal tensile stress at the top and bottom of the slab and differential deflections between slabs (nondoweled and doweled). As shown in Fig. 6, the corresponding value and position of the response is displayed by the interface in a way that is easy to interpret. When it is necessary to analyze multiple structures, the software has included a batch analysis module. Text files as shown in Fig. 7 can be imported and then analyzed. The results are tabulated on a comma-separated value file type (.csv) compatible with common commercial spreadsheet softwares.

Conclusions

The statistical calibration of these models is a step in a broad spectrum of investigation projects devoted to the local implementation of a mechanistic-empirical pavement design guide, which is

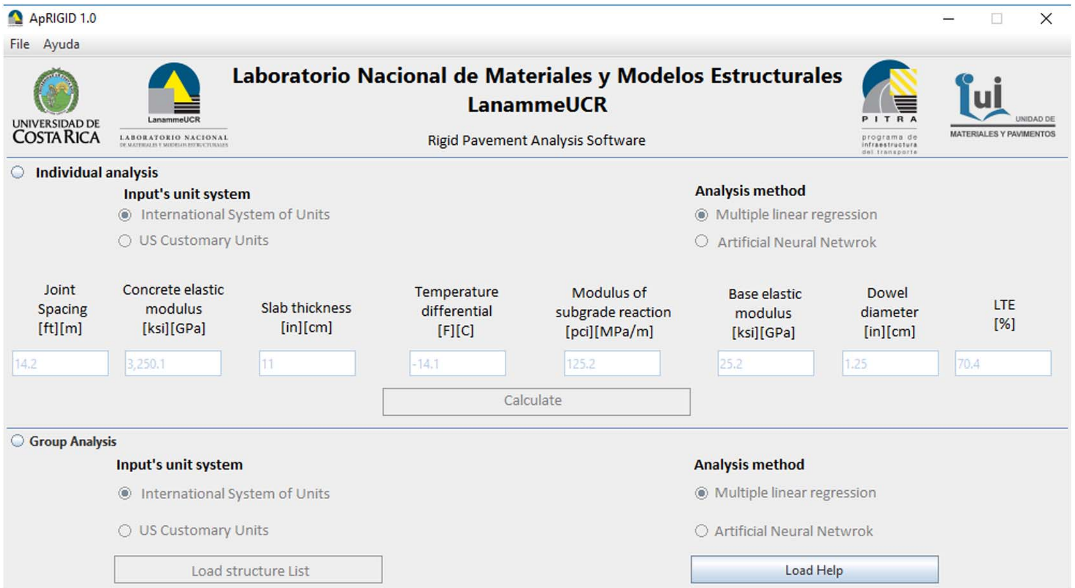

Fig. 5. *ApRIGID* graphical user interface—inputs

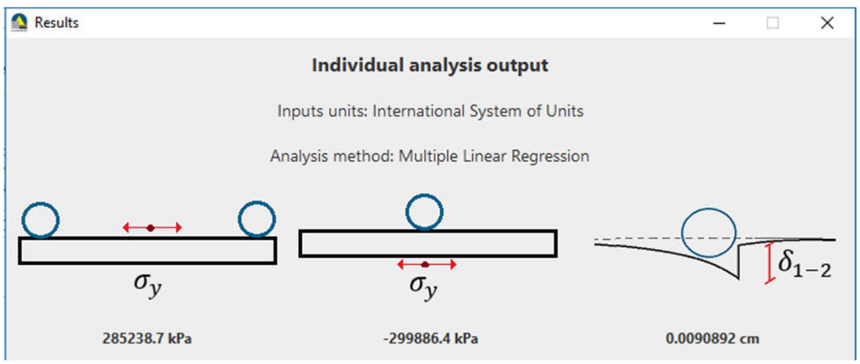

Fig. 6. *ApRIGID* graphical user interface—results

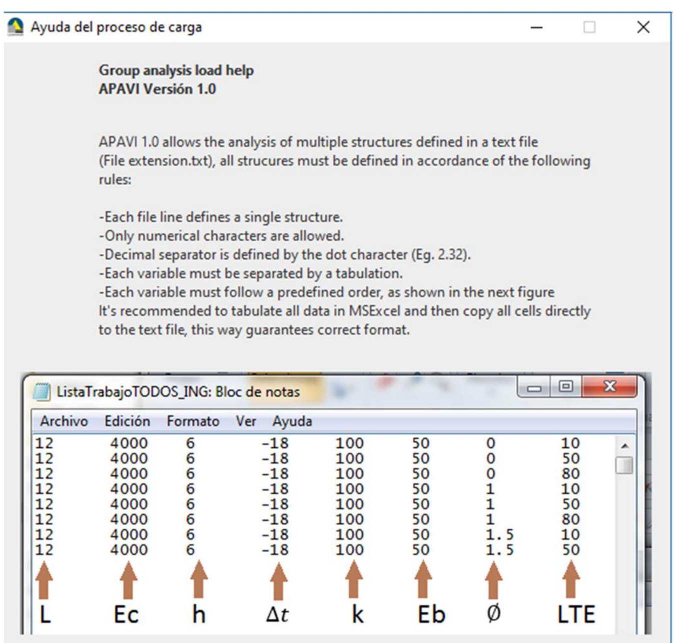

Fig. 7. *ApRIGID* graphical user interface—batch analysis module

conceptualized, calibrated, and validated for regional materials, service loads, climatic conditions, and construction techniques. The MLR structural models presented in this paper will be included in the initial guide draft as its structural analysis engine, allowing local designers a firsthand approach with the new design philosophy and an understanding of the underneath process, manual calculations, and advantages of such change from previous guides.

The calibration data set definition provided a large number of different structures, comprising most of the design possibilities available to local designers. The eight variable parameters are considered to be sufficient to characterize a regular JPCP structure and the materials used. Considering local variation of these parameters, the selected range of values was sufficient to include most of the possible design scenarios.

Model verification revealed that errors were concentrated on certain scenarios related with low tensile stresses of generally less than 210 kPa (30 psi). In some cases, critical compression stresses were observed in the data, in cases related with a positive interaction of load and temperature differentials effects. Evaluating a 210-kPa (30-psi) tensile stress in a fatigue model demonstrated that those scenarios are not to critical fatigue performance prediction and therefore negligible for all design purposes.

The verification data set served evaluation purposes in scenarios in which structures had certain parameters defined outside of the calibration ranges. The analysis showed that the models presented prediction problems with thin slabs of less than 10 cm (4 in.). Other than this, the models correctly extrapolated stress and deflection values for other scenarios, which included off-range elastic modulus, subgrade reaction values, temperature differentials, and dowel diameters. Thin JPCP slabs of less than 10 cm (4 in.) are not common in rigid pavement design, as their fatigue performance is poor. Thus, the limitation with these structures is not worth rejecting the calibrated models, as the models are perfectly capable for standard rigid pavement slab dimensions. It is possible that model improvement for these scenarios requires a different predictor and model design to adequately fit extreme stresses attributed to reduced slab thickness.

The artificial neural network models proved to be a viable method for critical response prediction. The results were satisfactory. It was possible to verify a better data fit, compared with multiple linear regression models, as the network training modeled better data nonlinearity and different interactions between parameters. A comparison between both methodologies showed a clear and substantial reduction of mean residual errors. Conversely, data extrapolation from calibration variable ranges is less reliable, which is something that must be noted for these models.

Considering all of this, the artificial neural network models here calibrated were recommended to be the structural engine for Level III (low-traffic rural roads, in which manual stress calculation is viable alongside basic material characterization) analysis of the proposed Costa Rican mechanistic-empirical pavement design guide. For Levels I and II (medium- and high-importance roads and more-specialized material characterization), artificial neural network models are recommended, as the use of design software is necessary. The software *ApRIGID 1.0* was developed as a tool for ANN models and their use for design purposes. Its architecture is highly modular, allowing the code to be reused for the future CR-ME design software. All software modules were thoroughly tested in local structures and proved to be an important tool for M-E implementation and training, allowing designers to analyze simultaneously a great number of pavement structures with minimal computing time.

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