Probabilistic Approach to Modeling Pavement Performance Using IRI Data

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Word Count:

Text	3802
Figures	4 x 250 = 1000
Tables	3 x 250 = 750
Total	5552

Abstract = 187 words

Paper Submitted for Presentation and Publication at the 93rd Annual Meeting of the Transportation Research Board

ABSTRACT

Accurately predicting pavement performance is an essential element in road infrastructure management. Pavement performance prediction methods can be either deterministic or probabilistic, depending on the method employed to simulate the deterioration or aging process. Deterministic models predict the condition on the basis of mathematical functions of observed or measured deterioration without taking uncertainties associated with the deterioration process into consideration. Probabilistic models, on the other hand, take uncertainties into consideration and predict the condition as the probability of occurrence in a range of possible outcomes. To overcome this shortcoming of deterministic models, probabilistic approaches have been investigated by various researchers. Probabilistic methods can be summarized into three categories: econometric models, Markov Chain Process (PMC) models, and reliability analysis. This paper presents a Markov Chain-based methodological framework to characterize pavement performance in support of pavement management decision makings. The International Roughness Index (IRI) data from the National DOT in the Costa Rican was used for the numerical case study to illustrate the application of the developed methodological framework. The findings from this study show that the proposed methodological framework is a viable approach to modeling pavement deterioration process.

INTRODUCTION

Pavement performance prediction is necessary for rational budget planning and resource allocation. At the network level, pavement performance prediction is needed for programming maintenance and rehabilitation (M&R) activities, while at the project level it is needed for determining the most appropriate M&R actions to be taken for a specific project, such as preventive maintenance, rehabilitation, or reconstruction (*1,2,3,4,5*).

Research shows it is less expensive to maintain a road than to repair it once it has significantly deteriorated (*2,3,4,5,6*). This is why pavement management systems (PMSs) prioritize corrective actions depending on pavement condition and available funds. If the system is in good condition, PMS would prioritize preventive maintenance rather than the reconstruction of roads in poor condition. Prioritizing the repair of bad sections of roadway over preventative maintenance for good roads is not the optimal use of funding resources. However, if current conditions of the network are fair or poor then preventive maintenance are not a PMS priority, and rehabilitation and reconstruction become the best alternative. In terms of life-cycle cost and long-term pavement conditions, following the aforementioned strategy results in better system performance (*1,3,4,5)*.

Accurate prediction of pavement performance requires reliable pavement deterioration models. Pavement performance prediction methods can be either deterministic or probabilistic, depending on the method employed to simulate the deterioration or aging process. Deterministic models predict the condition on the basis of mathematical functions of observed or measured deterioration without taking uncertainties associated with the deterioration process into consideration. Probabilistic models, on the other hand, take uncertainties into consideration and predict the condition as the probability of occurrence in a range of possible outcomes (*2,4,6,7,8,9,10)*.

Most deterministic models are based on establishing regression relationships between performance indicators and independent variables related to pavement performance, such as applied traffic loadings, material characteristics, and environmental conditions. Though deterministic models provide reasonably good prediction results, their deterministic nature does not allow them to be used to capture the inherent uncertainty in the process of pavement deterioration. In other words, these models are constrained by the fact that they cannot take the stochastic nature associated with the pavement performance into consideration *(2,4,5,9).*

To overcome this shortcoming of deterministic models, probabilistic approaches have been investigated by various researchers. Probabilistic methods can be summarized into three categories: econometric models, Markov Chain Process (MCP) models, and reliability analysis. Econometric models are widely used to correlate pavement distress with explanatory variables, such as structural number (SN), thickness of the surface layer, and number of wheel passes per unit strength of pavement (*2*). MCP is used to determine the transition from one state condition to another of a pavement section or network, using the Transition Probability Matrix *(4,6,8)*. The limitation associated with the traditional TPMs is that TPM cannot directly account for the impact of pavement types, environmental factors, traffic loading, and other relevant factors on the deterioration process. The improved econometric methods such as ordered probit model, Poisson model, and random-effects probit models were therefore proposed to connect the relevant explanatory variables with the transition probabilities. Time-based models, belonging to the reliability model category, are considered alternatives to Markov Chain models as they focus on estimating the probability distributions of time taken to transit from one condition state to another using duration models (*2,4,11)*.

The objective of study is develop a Markov Chain-based methodological framework to characterize pavement performance in support of pavement management decision makings. The International Roughness Index (IRI) data from the National DOT in the Costa Rican was used for the numerical case study to illustrate the application of the developed methodological framework. The rest of the paper is organized in four parts. First, a brief overview of the MCP is presented. Then, the proposed methodology is thoroughly explained. Next, a case of study using the primary road network in Costa Rica is discussed to demonstrate the use of the proposed methodology. Finally, the summary and findings are presented.

MARKOV CHAIN PROCESS (MCP)

In this study, MCP is used to determine the TPMs. The Markov prediction model is a stochastic process that: is discrete in time, has a countable or finite state space, and satisfies the Markov property. The Markov property is satisfied if the future state of the process depends on its present state, but not on its past states. In the pavements field, the Markov property is satisfied if the future condition of the network is dependent on the present condition of the network and not on its past condition *(8,13)*. In other words, the Markov Chain model has no memory of the past.

Road condition can be modeled by two types of Markov processes, homogeneous or non-homogeneous. In the case of the homogeneous process, the road network will always deteriorate following the transition probabilities of one single transition matrix. If the pattern of deterioration of a particular road network is likely to change at a certain point in time, *t*, the deterioration process may be modeled by a non-homogeneous process. This implies the use of a different transition matrix before and after *t*. In this case, the vector of the condition at *t* will become the starting vector for the second chain, operating with a different transition matrix. This type of arrangement may be performed as many times as required (*2*).

MCP models prove an effective method to predict performance deterioration of infrastructure facilities because of their ability to capture uncertainty of the deterioration process. Additionally, these models show an important applicability because of their relatively simple analytical procedure, becoming a very attractive alternative for DOTs in the U.S. and other highway agencies around the world, especially for network-level analysis (*4,6,10,11,12,14*). However, to analyze results correctly, model limitations should be considered. The memory-less property of MCP becomes one of the most important limitations because it using different TPMs at different times of analysis horizon implies that past conditions does affect future conditions. In addition, this approach does not provide a mechanism for physical factors important to the deterioration process to be incorporated in the modeling process. Physical factors can be introduced by econometric methods such as ordered probit model, Poisson model, and random-effects probit models. These models have been used successfully by various studies to calibrated TPMs for highway agencies. Despite modeling shortcomings the MCP provides a powerful relative easy methodology to develop pavement performance models at the network level.

METHODOLOGY

One of the important goals for highway agencies is to keep the highway infrastructure network in good condition with available funds. For this reason, pavement performance prediction becomes essential in order for decision makers to allocate funds as efficiently as possible. Figure 1 shows the proposed methodological framework for pavement performance modeling using MCP in the context of the generic infrastructure management process. The methodology consists of three main modules: data analysis, modeling, and results analysis.

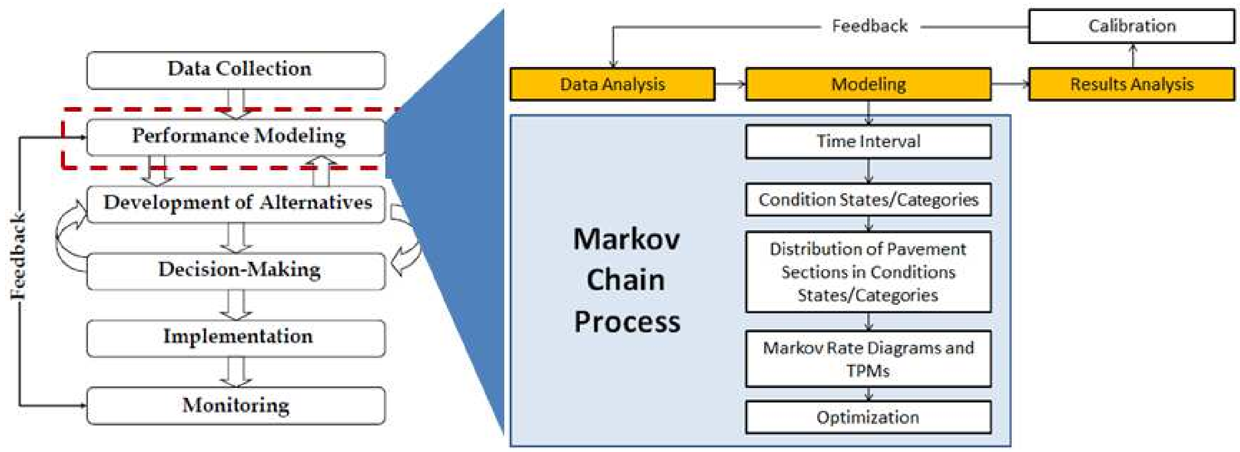


FIGURE 1 Methodology

Data Analysis

Data analysis serves as the first module of the proposed methodology. The objective is to determine the available information in the dataset that can be potentially used for pavement prediction modeling. The availability of such information as pavement structure, materials characterization, traffic volumes, climatic conditions, and M&R history should be determined by examining the existing datasets; if the information is available, it should be carefully analyzed in terms of its accuracy, format, completeness, integrity.

Modeling

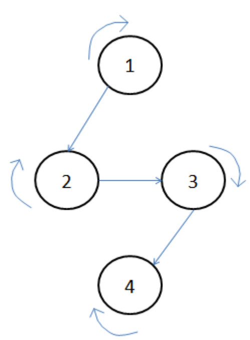
The second module is centered on modeling pavement deterioration using a probabilistic approach. More specifically, the proposed methodology is based on MCP, but tailored to satisfy the specific needs of this research, as illustrated in figure 1. The modeling module is divided into five steps: time interval determination, condition state formulation, distribution of pavement sections in the condition states, definition of the MRD and TPMs' computation, and optimization of the TPMs.

The first step in this module is to determine the time interval of the data collection, which in turn is used to define the time duration for the transition probability over which the pavement condition state will change from one cycle to the next. Most of the DOTs in the U.S. and certain highway agencies of other countries collect pavement condition data every one or two years. Usually, data collected can be classified in two categories: structural condition and functional condition, measuring the structural integrity and ride quality of pavement, respectively (*3,4,5*). It should be clarified that pavement condition is defined as a snapshot of the pavement structure in time *t*, while pavement performance is the trend of pavement condition over a period of time. The main objective of this study is to develop a probabilistic approach to predict pavement performance or the trend of pavement condition over time using available data.

After the time interval is defined, pavement condition indices and rating scores should be divided in a finite number of pavement condition states. The condition states should be carefully chosen such that they capture the full range of pavement behavioral conditions over the design life. More specifically, bands that bound condition states must be defined over a condition indicator if it is in rational scale, so that the probability of a pavement structure transitioning from one condition state to the next can be determined.

The third step is to distribute the network into condition states previously established, i.e., the condition of the network is depicted with the percentages of the total network in each condition state. Studies have shown that distribution can be presented in various attributes such as the number of pavement management sections, pavement lane miles, percentage of the road network, remaining service life, or distress measurements (*4,5,6,13*). Since the distribution defines the overall condition of the pavement network in each cycle being analyzed and, in turn, the TPMs, careful attention should be given to the selection of the attribute to be used to present the distribution.

The next step is to define the Markov Rate Diagram (MRD) considering the pavement condition states determined in previous steps. MRD is a graphical approach to understanding the transition of the pavement network from one condition state to the next. Additionally, it allows constraints to be incorporated to adjust for structural behavior of a pavement or to analyze different condition scenarios. For example, by setting the transition probability from condition states i to j to equal to 0, or $p_{ij} = 0$ for $i > j$, it implies that the pavement condition cannot be improved unless an M&R treatment is applied. Similarly, $p_{nn} = 0$ can be used to represent a holding condition state whereby where the pavement has reached its worst condition and cannot deteriorate further. Figure 2 shows an example of an MRD and its corresponding mathematical formulation in the form of a TPM. Each node represents the current condition and arrows indicate the transition rate when the pavement deteriorates from one state to the next. Additionally, the circular arrows indicate the probability that the pavement remains in the same condition state. After the MRDs are defined, the initial TPMs for each of the scenarios being considered can be developed.



$$P = \begin{bmatrix} p_{11} & p_{12} & 0 & 0 \\ 0 & p_{22} & p_{23} & 0 \\ 0 & 0 & p_{33} & p_{34} \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

FIGURE 2 Markov Rate Diagrams Example

The final step in this module is to optimize the TPMs by minimizing the error between the real road network distribution and the calculated distribution using the initial TPMs . The TPMs are optimized using the generalized reduced gradient nonlinear optimization code incorporated as an add-in to the software Microsoft Excel (*14*). The objective function used follows the form (*6*):

$$Z = min \sum_t \sum_i [a_t(i) - a_t'(i)]^2 \tag{1}$$

where:

$a_t(i)$ = ith term of the distributions obtained from the TPMs.

$a_t'(i)$ = ith element of the original data distribution obtained from step three

The objective function minimizes the difference between the distribution of pavements condition from the data set and the distributions obtained using the generated TPMs in the fourth step. From the definition of the objective function, it is obvious that the transition probability values or the elements of the TPMs are optimized through the optimization procedure (*6*).

Results Analysis

It is important to examine the probabilities values obtained in the TPMs to ensure that they correspond to the MRD. The next step is to use the TPMs to predict pavement performance in the following years to provide an insight on the M&R needs. Additionally, it is important to consider incorporating calibration procedures to adjust the TPMs when new data is collected. The new data obtained from new evaluation surveys will provide a feedback that should be used to further calibrate the TPMs.

CASE STUDY: COSTA RICAN ROAD NETWORK

The Costa Rican highway system consists of 22,258 miles (35,820 km), divided among municipal and national roads. A total of 17,398 miles (28,000 km) are part of the municipal roads network, while the other 4,859 miles (7,820 km) constitute the national roads network. Costa Rica has the second highest rate of roads by square kilometers in Latin America and is the number 52nd in the world with 72 km of roads per square kilometer area. From the national roads network, 60 percent are classified as flexible pavement, less than one percent is classified as rigid pavement, and gravel roads make up the remaining 39 percent. However, 98 percent of the budget used for maintenance and rehabilitation is invested in flexible pavement.

Data collection process

LanammeUCR has the ability to collect pavement performance data such as IRI, pavement deflections and pavement surface friction. However, the data set provided by LanammeUCR only comprises data related to the IRI and the FWD. Due to issues with the use of corrections for moisture and temperature, FWD data was discarded. Thus, the study utilizes pavement roughness as the only data to generate a probabilistic model for pavement performance for the Costa Rican primary road network. Moreover, the data collection process includes advanced data verification and quality control procedures. In Costa Rica, pavement roughness measurements are used for quality control and project acceptance. The largest application of IRI in the nation is the National Road Network Evaluation, which started in 2004 and is conducted every two years. The evaluations are performed using a Dynatest Inertial Profiler, Model 5051 Mark III Roadway Surface Profiler (RSP), property of LanammeUCR (*15*). The RSP computes the longitudinal and transverse profile, measures rutting, and registers the operational speed of the equipment.

Data analysis

For the evaluation of national road network, the results of the IRI are given at 100-m intervals, and reported as the average value of the IRI for the left and right sensors of the laser profiler. The average value of right and left IRI collected is known as the Mean Roughness Index (MRI). Additionally, the data is reported in millimeters/meters or meters/kilometers, as standardized by AASHTO R54 "Standard Practice for Accepting Ride Quality When Measured Using Inertial Profiling Systems". The data obtained is organized in shape files that can be manipulated in geographical information systems (GIS). Information such as the M&R zone, route, IRI from left and right wheel paths, and coordinates was found for each 100 meter section. Currently data is available for the following years: 2004, 2006, 2008, and 2010.

Pavement condition states

Because data obtained is collected in two-year cycles, for this study the time interval was defined as two years. Then the IRI data was discretized with the boundaries determined by using information from the National Road Network Evaluation Program, as shown in Table 1.

TABLE 1 Pavement Conditions State

Pavement Condition State	**IRI in in/mile (m/km)**
Good	0.00 – 70.87 (0.00 – 3.00)
Fair	70.87 – 106.30 (3.00 – 4.50)
Poor	106.30 – 151.18 (4.50 – 6.40)
Failed	>151.18 (>6.40)

Pavement condition for the years analyzed

The distribution of the pavement sections according to its IRI is computed in terms of percentage of the entire network. Each pavement section is classified with the boundaries specified in Table 1. Once the pavement sections are classified in each category, the percentages are calculated by dividing the total network's length of each category by the total length of the network evaluated. Table 2 shows the percentage of the road network for each year distributed in the four condition states previously established.

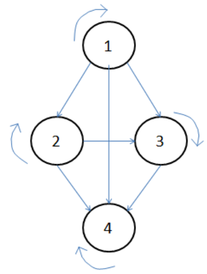
It can be seen from Table 2 that the pavement network experiences degradation in terms of IRI from 2004 to 2010. The "Good" conditions percentage is reduced by 8 percent, while the "Fair" condition increased by approximately 8 percent. There is a reduction in "Poor" condition. However, the "Failed" condition increases by almost 5 percent. Since maintenance and rehabilitation information is not available, all sections were considered in calculating the network distribution.

TABLE 2 National Road Network states condition

Year	Condition (percent)				Control Sections	Network Length (km)
	Good	Fair	Poor	Failed		
2004	30	28	28	14	539	3460
2006	26	30	25	19	780	4365
2008	28	32	24	16	780	4365
2010	22	35	23	20	780	4365

MRDs and TPMs

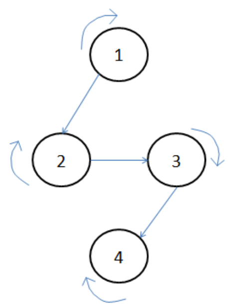
The Markov Rate Diagrams (MRDs) were defined by considering pavement conditions states shown in Table 1. In order to analyze different conditions for the road network, various Markov diagrams are generated. As examples, four diagrams are shown in figure 3. After distributing the road network into established categories, the initial TPMs for each case under consideration are computed. Using data from years 2004 and 2006, a preliminary TPM was computed for each of the four rate diagrams cases shown in Figure 3.



$$P = \begin{bmatrix} p_{11} & p_{12} & p_{13} & p_{14} \\ 0 & p_{22} & p_{23} & p_{24} \\ 0 & 0 & p_{33} & p_{34} \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

For this condition all transitions moving forward can occur, meaning that $i \leq j$ has to be true for all p_{ij}. The transitions backwards are restrained. The cell with zero values means that there is no probability of moving from a higher state condition to a lower condition state. The last assumption considers that a road cannot improve itself without first receiving treatment.

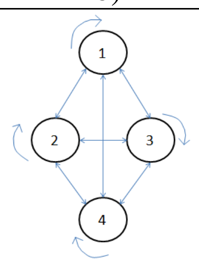
a)



$$P = \begin{bmatrix} p_{11} & p_{12} & 0 & 0 \\ 0 & p_{22} & p_{23} & 0 \\ 0 & 0 & p_{33} & p_{34} \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

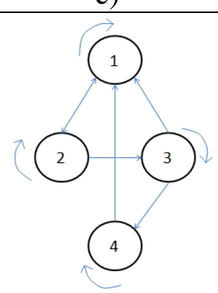
In this case, a further restriction was added to the analysis. The control sections were allowed to deteriorate by no more than one state in each cycle. This configuration in pavement deterioration modeling is commonly used because it is assumed that degradation follows the natural sequence, moving from one state to another rather than moving more than one state at the time (*3,7,8,10*). For the two configurations previously discussed a steady-state condition was not expected since $j < i$ for all entries p_{ij} were restricted to be equal to zero.

b)



$$P = \begin{bmatrix} p_{11} & p_{12} & p_{13} & p_{14} \\ p_{21} & p_{22} & p_{23} & p_{24} \\ p_{31} & p_{32} & p_{33} & p_{34} \\ p_{41} & p_{42} & p_{43} & p_{44} \end{bmatrix}$$

c)



$$P = \begin{bmatrix} p_{11} & p_{12} & 0 & 0 \\ p_{21} & p_{22} & p_{23} & 0 \\ p_{31} & 0 & p_{33} & p_{34} \\ p_{41} & 0 & 0 & p_{44} \end{bmatrix}$$

d)

The next two rate diagrams were designed to include repair actions. Information regarding specific repairs for each section was not available. Therefore, special treatment analysis was not carried in this study.

For (c), all transition probabilities are assumed to be possible as shown in the matrix form (all entries are allowed).

The last rate diagram (d) shows the degradation condition explained previously for the second configuration. However, repair actions were introduced. In this design, the repair is assumed to move any condition state back to the best condition state, signifying that maintenance actions will recover the pavement performance in terms of the IRI to a good condition.

FIGURE 3 Markov Chain Rate Diagrams (a) MRD case 1, (b) MRD case 2, (c) MRD case 3 and (d) MRD case 4

TMP Optimization

The TPMs were optimized by minimizing the error between the real road network distribution and the calculated distribution using the initial TPMs. The four estimated TPMs are optimized using the generalized reduced gradient nonlinear optimization code incorporated as an add-in to software Microsoft Excel (*14*) and the formulation was discussed earlier in the Modeling section.

RESULTS

This section divides the results in two categories: calculated TPMs and prediction for the year 2020.

Calculated TPMs

TPMs were derived for each of the previously discussed four cases, as shown in Table 3. TPM MRD cases 1 and 2 show the results for the cases without M&R treatments applied, as the pavement sections showing improvements in condition were dropped from the dataset. However, the pavements sections that stayed in the same condition state were included in the dataset regardless if M&R treatments were applied. The lower two matrices illustrate the results when improvements were observed, which implies that M&R treatments were applied to the pavement sections.

TABLE 3 Calculated TPMs

	Good	Fair	Poor	Failed
Good	0.850	0.150	0.000	0.000
Fair	0.000	0.800	0.127	0.073
Poor	0.000	0.000	0.773	0.227
Failed	0.000	0.000	0.000	1.000

TPM MRD case 1

	Good	Fair	Poor	Failed
Good	0.851	0.149	0.000	0.000
Fair	0.000	0.860	0.140	0.000
Poor	0.000	0.000	0.847	0.153
Failed	0.000	0.000	0.000	1.000

TPM MRD case 2

	Good	Fair	Poor	Failed
Good	0.712	0.077	0.031	0.179
Fair	0.127	0.809	0.064	0.000
Poor	0.036	0.139	0.737	0.088
Failed	0.002	0.163	0.138	0.697

TPM MRD case 3

	Good	Fair	Poor	Failed
Good	0.756	0.244	0.000	0.000
Fair	0.000	0.863	0.137	0.000
Poor	0.134	0.000	0.771	0.095
Failed	0.028	0.000	0.000	0.972

TPM MRD case 4

As expected, the matrix diagonal and the offset diagonal show the highest values for the MRD cases 1 and 2. As briefly discussed earlier, MRD case 3 is a scenario where pavement sections with M&R treatments are included. The MRD case 4 assumes that M&R would only improve the current condition state “Good” (condition state 1).

Since neither pavement management system nor pavement prediction models are currently being used by transportation authorities in Costa Rica, TPMs could provide a decision-support tool to allocate funds using a data-driven approach, instead of an experienced-based subjective approach. Though the developed TPMs need to include more pavement performance indicators; however, they can serve as an initial step towards more data-driven pavement management systems for Costa Rica.

Costa Rican Road Network 2020 Predictions

Based on the calculated TPMs, the state conditions of the network for year 2020 are predicted. The results are presented in pie plots for a more comprehensive comparison among the different studied conditions. Figure 4 illustrates the predictions obtained.

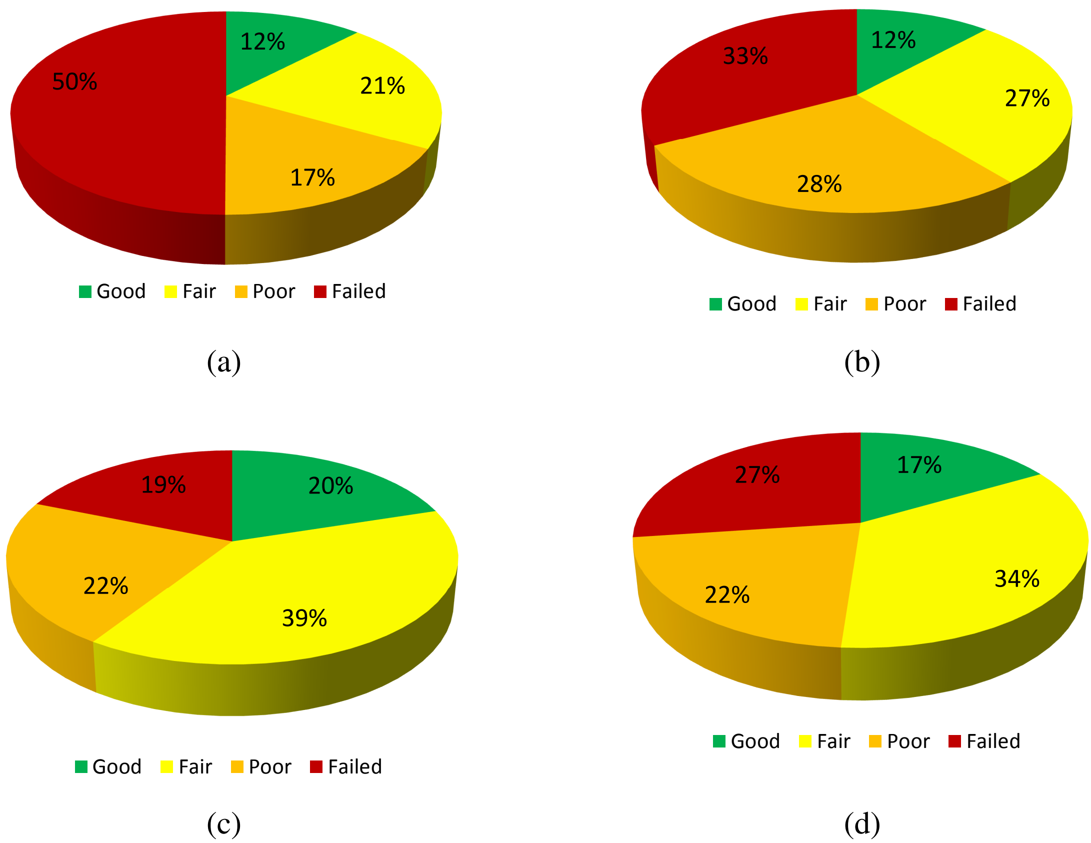


FIGURE 4 2020 Pavement Performance Predictions
(a) MRD case 1, (b) MRD case 2, (c) MRD case 3 and (d) MRD case 4

For the two categories analyzed, with and without repair, it shows that results for the year 2020 are similar. The matrices for the no-repair condition are similar in terms of magnitude of each of the probabilities. Consequently, the percentages in each condition state for 2020 show the same distribution. The same was observed when repair is added in the analysis. However, when repair is incorporated in analysis, the failed condition percentage is reduced by almost 10 percent. This reduction indicates the importance of considering M&R programs to maintain a certain level within budget constraints.

CONCLUSIONS

The overall objective of this paper is to develop a Markov Chain-based methodological framework to characterize pavement performance in support of pavement management decision makings. The methodology proposed was successfully utilized for the case of study. Major conclusions drawn from this study include:

- The dynamic nature of the proposed TPMs can be effectively used for pavement deteriorations modeling. Uncertainties related to pavement performance can be taken into consideration using the proposed TPMs, allowing pavement condition to be predicted as the probability of being in one of the pre-defined condition states.
- Optimization can be used to minimize the errors associated with TMPs. By comparing the distribution of pavements condition from the data set and that obtained using the generated TPMs, the errors can be significantly reduced by using the proposed optimization techniques. As illustrated in the example, the TPMs closely follow the data set after the optimization is performed, resulting in more reliable TPMs and, in turn, more accurate prediction of pavement performance.
- The proposed framework, as demonstrated by the case study, can be applied to a wide range of conditions by various highway agencies. It provides a relative easy methodology for pavement deterioration modeling that could enhance the decision-making process in highway agencies with limited pavement data, especially at the network level.

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