

1           **EVALUATION OF FATIGUE LIFE OF ASPHALT MIXTURES THROUGH THE**  
2           **DISSIPATED ENERGY APPROACH**

3  
4  
5           *Submitted to the 94th Annual Meeting of the Transportation Research Board*  
6           Submitted on July 21, 2014

7  
8  
9                           Adriana Vargas-Nordbeck, Ph.D.  
10                           (Corresponding author)  
11           National Laboratory of Materials and Structural Models (LanammeUCR)  
12                           University of Costa Rica, San José, Costa Rica  
13                           adriana.vargasnordbeck@ucr.ac.cr

14  
15                           José Pablo Aguiar-Moya, Ph.D.  
16           National Laboratory of Materials and Structural Models (LanammeUCR)  
17                           University of Costa Rica, San José, Costa Rica  
18                           jose.aguiar@ucr.ac.cr

19  
20                           Fabricio Leiva-Villacorta, Ph.D.  
21           National Laboratory of Materials and Structural Models (LanammeUCR)  
22                           University of Costa Rica, San José, Costa Rica  
23                           E-mail: fabricio.leiva@ucr.ac.cr

24  
25                           Luis Loria-Salazar, Ph.D.  
26                           Pavement Infrastructure Program Director  
27           National Laboratory of Materials and Structural Models (LanammeUCR)  
28           University of Costa Rica, P.O.Box 11501-2060, UCR, San José, Costa Rica  
29                           E-mail: luis.loriasalazar@ucr.ac.cr

30  
31  
32   Word Count: Abstract (240) + Body (3,346) + Figures and Tables (15 \* 250) = 7,336

**1 ABSTRACT**

2  
3 Fatigue cracking constitutes a main type of distress for flexible pavement, and therefore  
4 developing adequate fatigue models is one of the key challenges in the mechanistic-empirical  
5 design method. One of the most popular test procedures used to determine susceptibility to  
6 fatigue cracking in the laboratory is the beam flexural test. The results are usually interpreted in  
7 terms of a relationship between applied stress or strain and number of cycles to failure. Although  
8 this phenomenological approach provides some guidance necessary to understand fatigue  
9 performance of asphalt concrete pavements, it is essentially an empirical approach that requires  
10 continuous calibration since the relationship between the parameters is not unique and depends  
11 on material properties and loading mode, among others. The dissipated energy approach is based  
12 on the determination of the plateau value (PV), a fundamental property which has a unique  
13 relationship with the fatigue life of asphalt mixtures. The main objective of this research was to  
14 evaluate the dissipated energy approach as an alternative to assess the fatigue life of asphalt  
15 mixtures in Costa Rica. This study used historical data from the beam flexural fatigue test to  
16 validate the relationship between the plateau value and the number of cycles to failure and  
17 evaluated several models for the prediction of PV. The results showed that the dissipated energy  
18 approach is a more accurate alternative for fatigue analysis and the models developed can  
19 eventually be applied to pavement design without the need for extensive testing.

## 1 INTRODUCTION

2  
3 Fatigue cracking is one of the main distress types for flexible pavements. Cracks generally  
4 initiate at the bottom of the asphalt layer due to the large tensile strains produced by repetitive  
5 traffic loads and propagate upwards as the loadings continue, eventually appearing on the surface.  
6 However, load-related fatigue cracking can also initiate at or near the surface of the pavement  
7 and propagate from the top down (1).

8 Fatigue cracking resistance of asphalt mixtures depends on material properties as well as  
9 pavement structural factors. In the laboratory, fatigue evaluation is focused on factors related to  
10 the material properties of the hot mix asphalt (HMA) mixtures. One of the most popular test  
11 procedures used to determine susceptibility to fatigue cracking is the beam flexural test. This test  
12 was designed to simulate the bending that a HMA layer experiences in a pavement structure. The  
13 results are usually interpreted in terms of a relationship between applied stress or strain and  
14 number of cycles to failure (2, 3). There are several models used to predict the fatigue life of  
15 asphalt mixtures, the simplest one being the model proposed by Pell (4). For a controlled-strain  
16 test, the relationship is described by Equation 1:  
17

$$N_f = k_1 \left( \frac{1}{\varepsilon} \right)^{k_2} \quad (1)$$

18 where

19  $N_f$  = number of cycles to failure

20  $\varepsilon$  = tensile strain, mm/mm

21  $k_1, k_2$  = mix-dependent regression coefficients

22  
23 Although this phenomenological approach provides some guidance necessary to  
24 understand fatigue performance of HMA pavements, there are limitations that must be  
25 considered. It is essentially an empirical approach and does not provide a relationship between  
26 loading and any form of damage accumulation in the mixture (5). The results are either material  
27 dependent, or loading mode dependent, or both, so this approach cannot be applied directly to the  
28 complex loading scenarios that are actually common to in-service pavements (6). In addition, the  
29 strain fatigue life relationship is treated linearly, which has been found to be inappropriate at low  
30 strains (7).  
31

## 32 Dissipated Energy Approach

33  
34 Dissipated energy is a measure of the energy that is lost to the material or altered through  
35 mechanical work, heat generation, or damage to the sample (8). Ghuzlan and Carpenter (9)  
36 developed the ratio of dissipated energy change (RDEC) concept to define fatigue failure in  
37 asphalt mixtures. This approach considers only the portion of the dissipated energy that is  
38 responsible for actual damage. The RDEC is defined as the change in dissipated energy between  
39 two cycles divided by the dissipated energy of the first cycle, as shown in Equation 2.  
40

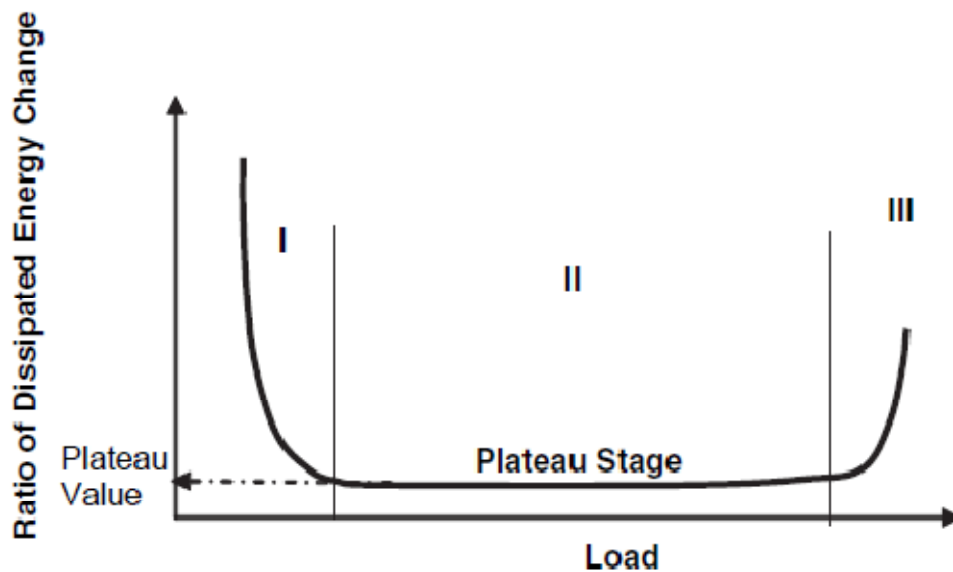
$$RDEC = \frac{DE_{n+1} - DE_n}{DE_n} \quad (2)$$

41 where

- 1 RDEC = ratio of dissipated energy change  
 2  $DE_n$  = dissipated energy produced in load cycle  $n$   
 3  $DE_{n+1}$  = dissipated energy produced in load cycle  $n+1$   
 4

5 The damage curve represented by RDEC versus the number of loading cycles can be  
 6 divided into three regions (6, 7, 9), as shown in Figure 1. Region I corresponds to the initial  
 7 "settling" of the sample where the RDEC decreases rapidly. In Region II, the RDEC reaches a  
 8 plateau during which a constant portion of energy is being turned into damage. In Region III, the  
 9 rapid increase in RDEC indicates sample instability and is the onset of true failure.

10 The nearly constant value of RDEC in Region II is defined as the plateau value (PV). The  
 11 PV is proposed as a fundamental damage parameter that provides a unique relationship with  
 12 fatigue life for different mixtures, loading modes and loading levels (6, 9).  
 13



14 **FIGURE 1** Typical dissipated energy ratio plot with three behavior zones (6).

## 17 OBJECTIVE

18  
 19 The main objective of this study was to evaluate the dissipated energy approach as an alternative  
 20 to assess the fatigue life of asphalt mixtures in Costa Rica.

## 22 SCOPE OF WORK

23  
 24 To accomplish the aforementioned objective, historical data from the beam flexural test were  
 25 used. Tests were conducted in accordance with AASHTO T 321 (10) under constant strain  
 26 loading for strain levels ranging from 200 to 800  $\mu\epsilon$ . A total of 617 raw data files were collected  
 27 from tests performed between 2004 and 2013, which included laboratory and plant produced  
 28 mixtures. Over this period, different criteria have been used to perform the test, such as curing  
 29 time of the specimens or required air void content. These discrepancies have made it difficult to  
 30 develop a reliable general fatigue model based on the phenomenological approach because they  
 31 increase variability. The results also included mixtures not typically used in Costa Rica, such as

1 stone matrix asphalt (SMA), open-graded friction courses (OGFC) and polymer modified  
 2 mixtures. When possible, additional information regarding mixture properties and results from  
 3 other laboratory tests were also obtained, but this was not available for all samples.

4 To ensure the validity of the data, an extensive quality control procedure was conducted.  
 5 Each individual data file was checked to verify the failure criteria of 50 percent reduction in  
 6 stiffness from initial stiffness. Additionally, the dissipated energy curve was inspected for every  
 7 specimen and results containing erratic curves were discarded.

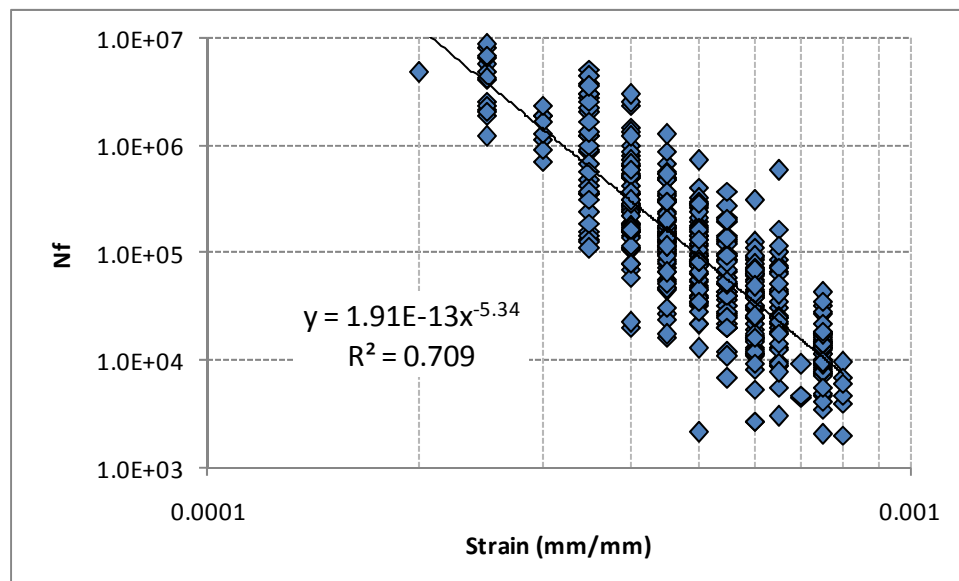
8 Test results were used to calculate the plateau value of each specimen. A detailed  
 9 description on the procedure followed to calculate the PV can be found elsewhere (6, 9). This  
 10 study is based only on results from tests performed at 20°C.

## 12 RESULTS AND DISCUSSION

### 14 Phenomenological Approach

16 The traditional phenomenological analysis was used to establish a relationship between strain  
 17 level and the number of cycles to failure ( $N_f$ ). Although this relationship is mixture dependent,  
 18 results from all samples were pooled together to obtain the general model in Equation 3. Figure 2  
 19 shows the fitted curve for all mixtures. It can be observed that there is a well defined trend  
 20 between both parameters; however, significant data scatter is also present.

$$N_f = 1.91 \times 10^{-13} \left( \frac{1}{\varepsilon} \right)^{5.34} \quad (3)$$

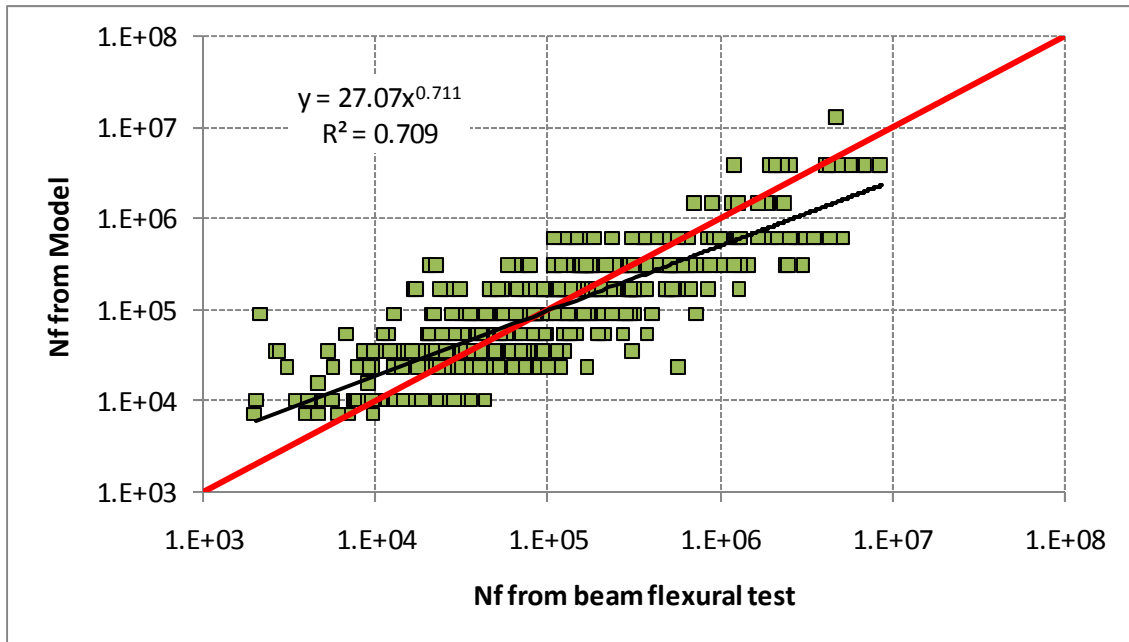


23 **FIGURE 2**  $N_f$  - strain relationship for all mixtures.

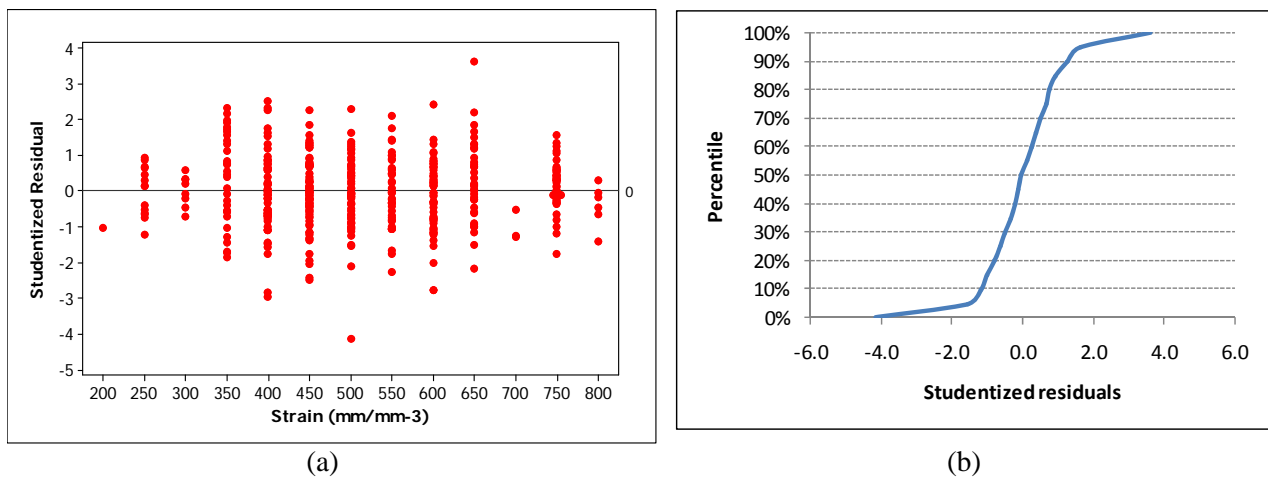
24  
 25  
 26 Figure 3 shows measured versus predicted values of  $N_f$ . It is evident that the model  
 27 developed tends to overestimate the fatigue life at low number of cycles (generally corresponding  
 28 to high strain levels), while at higher number of cycles (low strain levels) the  $N_f$  values are  
 29 mostly underestimated. This is reflected by the best-fit line coefficients obtained for the

1 relationship between measured and predicted number of cycles to failure. When these coefficients  
 2 are close to 1, it is an indication that the values are on average close to the line of equality.  
 3 However, in this case the larger intercept and lower slope imply that the calculated  $N_f$  deviate  
 4 from the measured fatigue lives, particularly at both ends of the range of values studied.  
 5

6 The plot shown in Figure 4a indicates that the studentized residuals from the regression (a  
 7 scaled measure of error) are in general evenly distributed for each strain level. A few extreme  
 8 values were identified, but as illustrated in Figure 4b, the majority of the studentized residuals  
 9 (approximately 96% of the data) are within -3 and 3.  
 10



11 **FIGURE 3** Comparison between measured and calculated  $N_f$  results using Equation 3.  
 12  
 13  
 14



15 **FIGURE 4** Residual analysis for phenomenological model in Equation 3: a) Residuals vs. strain,  
 16 b) Cumulative distribution plot of residuals  
 17

1 Some researchers have suggested that a relationship more applicable to asphalt mixtures  
 2 in general should include stiffness as a variable to account for the differences in mixture  
 3 properties (11). This model form was also developed for the samples included in this study,  
 4 resulting in Equation 4.

$$N_f = 1.64 \times 10^{-12} \left(\frac{1}{\varepsilon}\right)^{5.34} \left(\frac{1}{S}\right)^{0.24} \quad (4)$$

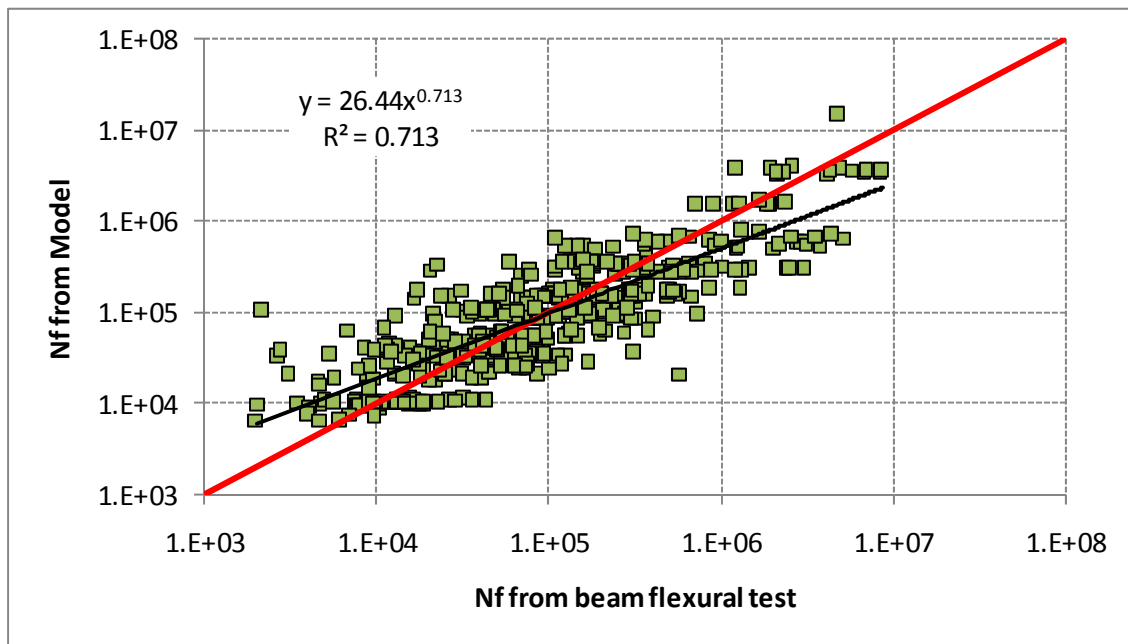
6 where

7  $N_f$  = number of cycles to failure

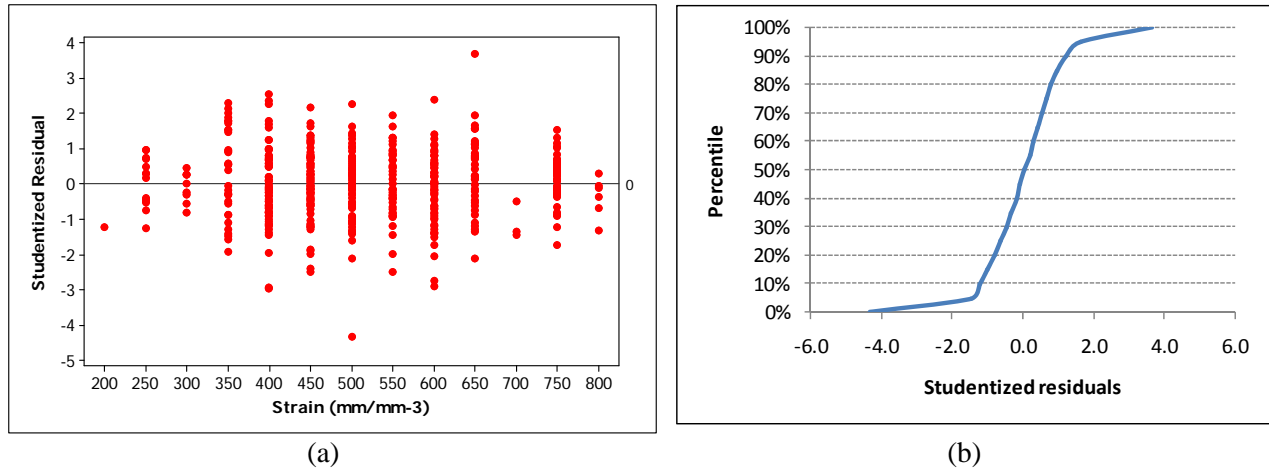
8  $\varepsilon$  = tensile strain, mm/mm

9  $S$  = initial mix stiffness, MPa

10  
 11 Figure 5 shows a comparison of measured and predicted  $N_f$  values using Equation 4. It  
 12 can be observed that including the stiffness term in the model did not have a significant impact  
 13 with respect to the trends found using Equation 3. Fatigue lives are still overestimated in some  
 14 cases and underestimated in others, while the distribution of residuals (Figure 6) is essentially the  
 15 same. These results suggest that the phenomenological approach may not be appropriate for  
 16 developing general models, or it may require extensive calibration to reduce error.



18 **FIGURE 5.** Comparison between measured and calculated  $N_f$  results using Equation 4.  
 19  
 20



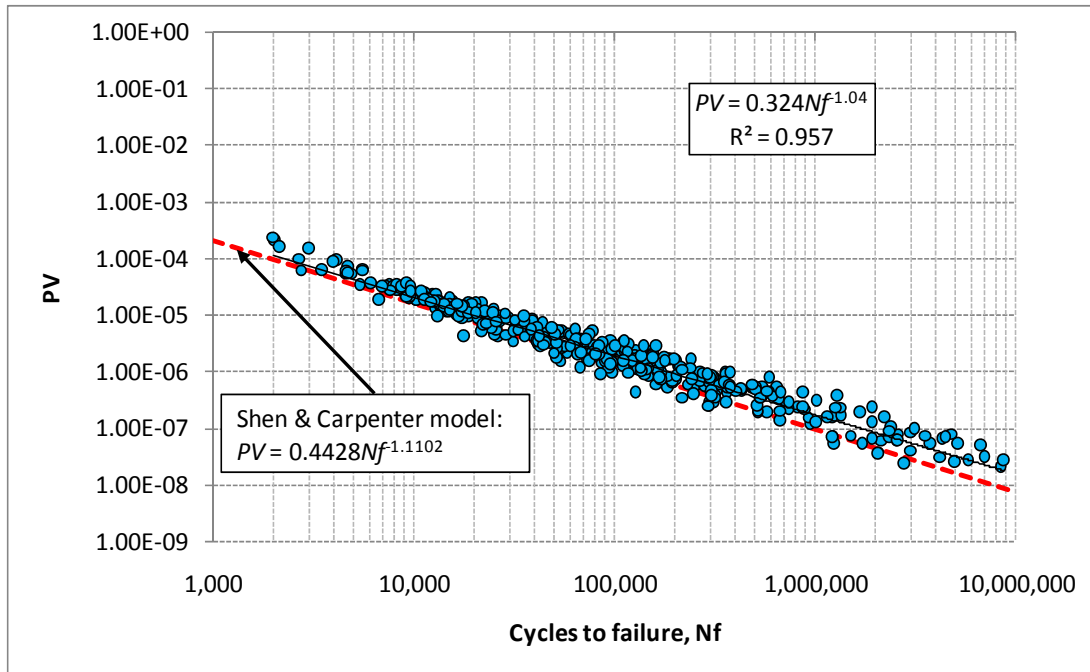
1 **FIGURE 6** Residual analysis for phenomenological model in Equation 4: a) Residuals vs. strain,  
 2 b) Cumulative distribution plot of residuals  
 3

#### 4 **PV - $N_f$ Relationship**

5  
 6 Although Equations 3 and 4 capture the general fatigue behavior of asphalt mixtures, it is  
 7 possible to obtain a fatigue model with less variability by using an approach that is independent  
 8 of mixture properties and loading conditions. The model shown in Equation 5 was obtained to  
 9 describe the relationship between the plateau value (PV) and the number of cycles to failure ( $N_f$ ).  
 10 This relationship, illustrated in Figure 7, was very similar to the exponential equation developed  
 11 by Shen and Carpenter (6) but had slightly lower regression coefficients (intercept and slope). For  
 12 the mixtures included in this study, a higher variability was observed for the results as the fatigue  
 13 life increased.  
 14

$$15 \quad PV = 0.324N_f^{-1.04} \quad (5)$$





**FIGURE 7** PV -  $N_f$  relationship for Costa Rican mixtures at 20°C.

Compared to the phenomenological approach, it is evident that the dissipated energy-based model exhibits less variability and is more appropriate for predicting fatigue life when including all mixtures in the database. Table 1 shows a comparison of the model parameters using both approaches. The PV- $N_f$  model had a higher coefficient of determination ( $R^2$ ) and a lower residual mean square ( $MS_{RES}$ ), indicating a better fit of the data.

**TABLE 1** Comparison of Analysis Approaches for Assessing Fatigue Life

Approach	Model	$R^2$	$MS_{RES}$
Phenomenological	$N_f = 1.91 \times 10^{-13} \left(\frac{1}{\varepsilon}\right)^{5.34}$	0.709	0.1633
Phenomenological	$N_f = 1.64 \times 10^{-12} \left(\frac{1}{\varepsilon}\right)^{5.34} \left(\frac{1}{S}\right)^{0.24}$	0.713	0.1579
Dissipated energy	$PV = 0.324N_f^{-1.04}$	0.957	0.0265

### PV Prediction Model

Once a relationship was established between the PV and the fatigue life of the asphalt mixtures, the study focused on obtaining a model to predict the plateau value without the need for extensive testing. Shen and Carpenter (6) proposed an equation based on load effect and material properties, as shown in Equation 6.

$$PV = 44.422\varepsilon^{5.140}S^{2.993}VP^{1.850}GP^{-0.4063} \quad (6)$$

where

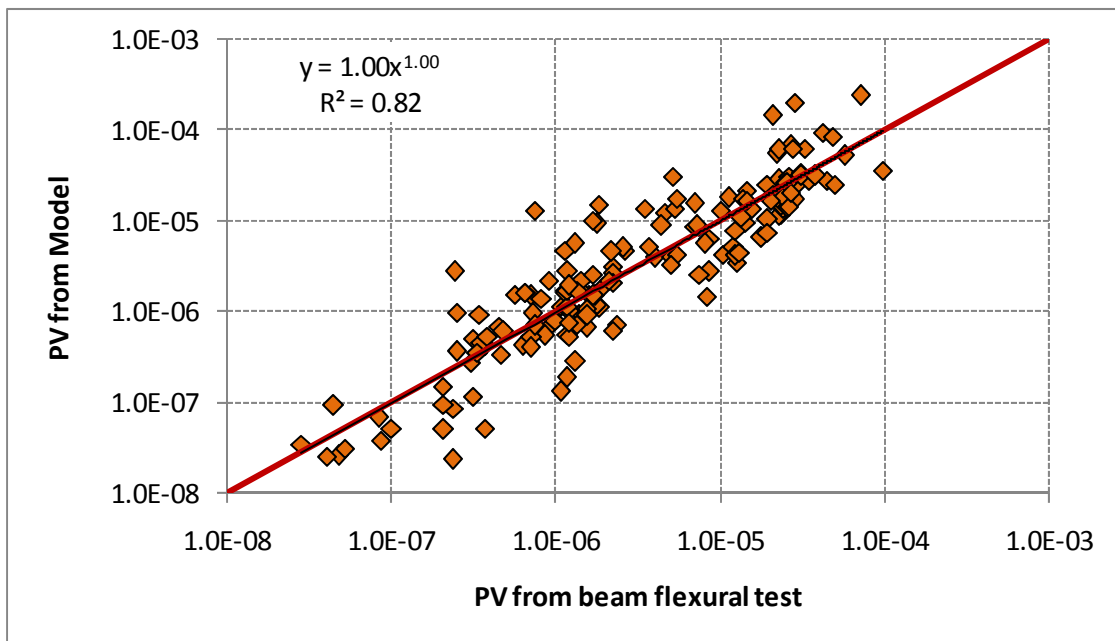
$\varepsilon$  = tensile strain, in/in

- 1 S = flexural stiffness of the mixture (20°C, 10Hz), MPa  
 2 VP = volumetric parameter ,  $VP = \frac{AV}{AV+V_b}$   
 3 AV = mixture air voids, %  
 4  $V_b =$  mixture asphalt content by volume, %,  $V_b = 100 \times \frac{G_{mb} \times P_b}{G_b}$   
 5  $G_{mb}$  = mixture bulk specific gravity  
 6  $P_b$  = percent of asphalt binder by total weight of mix, %  
 7  $G_b$  = asphalt binder specific gravity (generally assumed 1.03)  
 8 GP = aggregate gradation parameter,  $GP = \frac{P_{NMQS} - P_{PCS}}{P_{200}}$   
 9  $P_{NMQS}$  = percent of aggregate passing the nominal maximum size sieve, %  
 10  $P_{PCS}$  = percent of aggregate passing the primary control size sieve, %  
 11  $P_{200}$  = percent of aggregate passing the No. 200 sieve, %  
 12

13 Using the same parameters, a new model was calibrated for Costa Rican mixtures. In this  
 14 case, a reduced database was used since as mentioned earlier, volumetric information was not  
 15 available for all samples. The model obtained is shown in Equation 7.  
 16

$$PV = 10^{9.505} \varepsilon^{6.0612} S^{1.5091} VP^{1.4684} \quad (7)$$

17 The regression analysis determined that the gradation parameter was not statistically  
 18 significant, so this term does not appear in the calibrated model. Figure 8 shows a comparison  
 19 between measured and calculated plateau values. On average, results fall along the line of  
 20 equality. Although the model shows good correlation with laboratory data, Equation 7 would still  
 21 require the beam flexural test to be performed in order to obtain the stiffness of the mix (S).  
 22



23  
 24 **FIGURE 8** Comparison between measured and calculated PV results.  
 25

26 To simplify the evaluation of the fatigue life of asphalt mixtures, other tests may be used  
 27 to characterize the stiffness of the mixes. Consequently, other models that included variables such

1 as dynamic modulus ( $E^*$ ) and phase angle ( $\phi$ ), resilient modulus ( $M_R$ ) and tensile strength ( $S_t$ )  
 2 were studied. Table 2 shows the different models considered in this study along with their  
 3 corresponding parameters ( $R^2$ ,  $R_{adj}^2$  and  $MS_{RES}$ ). The stepwise regression procedure was used to  
 4 select the regressor variables in each equation. It can be observed that all models exhibited high  
 5  $R^2$  and  $R_{adj}^2$  values, which indicate that a high portion of the variability observed in the PV is  
 6 explained by the model.

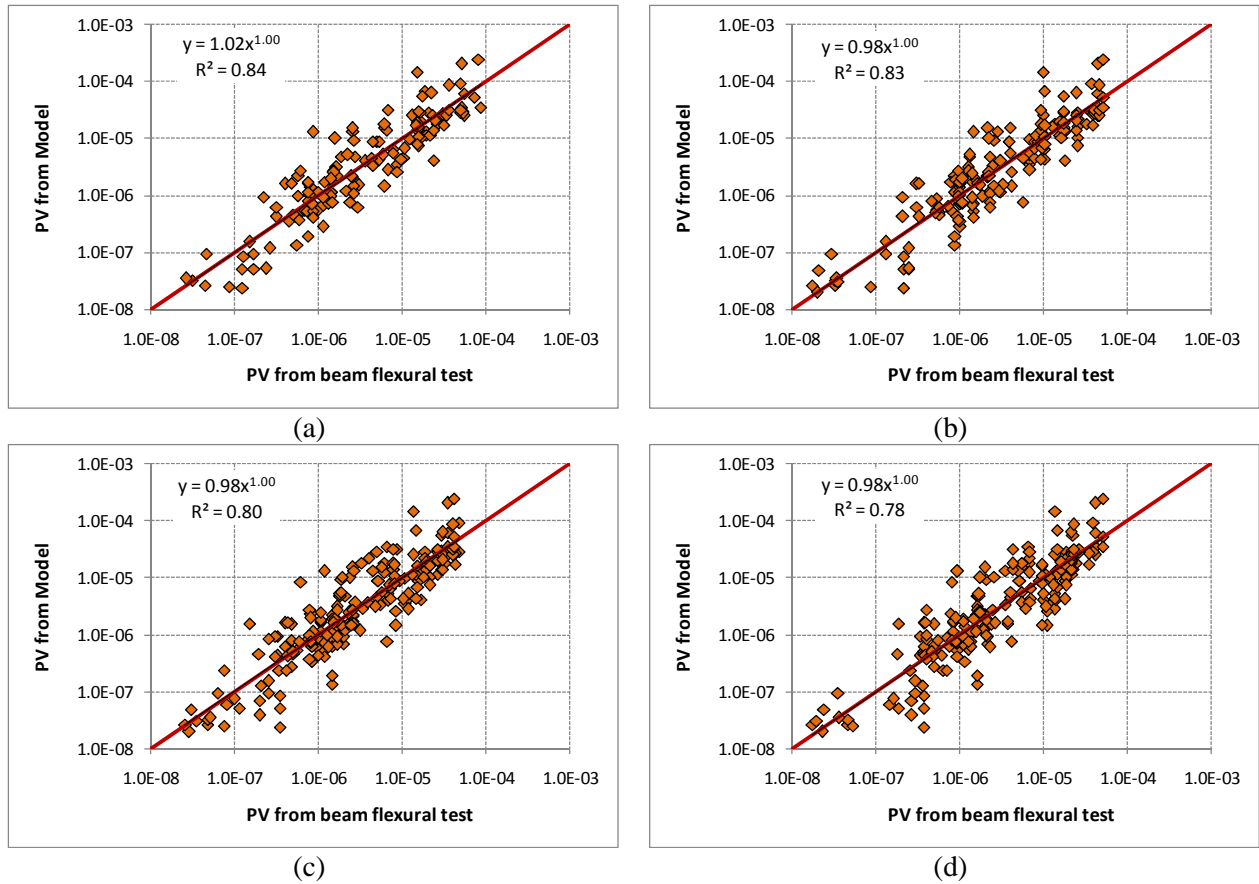
7  
 8 **TABLE 2** Models Developed for PV Prediction

Eq.	Variables	Fitted model	$R^2$	$R_{adj}^2$	$MS_{RES}$
8	$\varepsilon$ , $E^*$ , VP, GP	$PV = 5.6 \times 10^{-4} \varepsilon^{5.8268} E^{4.7652} VP^{0.7341} GP^{-1.1644}$	0.837	0.833	0.132
9	$\varepsilon$ , $E^*$ , $\phi$	$PV = 10^{7.426} \varepsilon^{5.5806} E^{2.3163} \phi^{-2.7170}$	0.835	0.832	0.135
10	$\varepsilon$ , $M_R$ , VP, GP	$PV = 10^{8.415} \varepsilon^{5.6690} M_R^{1.2663}$	0.802	0.800	0.150
11	$\varepsilon$ , $S_t$	$PV = 10^{8.365} \varepsilon^{5.8175} S_t^{1.7278}$	0.785	0.783	0.163

9  
 10 Equations 8 and 9 used the dynamic modulus of the mixtures measured at 20°C and 10 Hz  
 11 as a predictor for PV. The former includes volumetric and gradation parameters, while the latter  
 12 estimates the PV based only on the dynamic modulus and phase angle. Although Equation 8 has  
 13 slightly better regression parameters, Equation 8 can provide a good approximation of the plateau  
 14 value while requiring less information. In Equation 10, the resilient modulus accounts for the  
 15 stiffness of the mixture. The volumetric and gradation parameters were initially considered in the  
 16 model, but were not found to be significant.

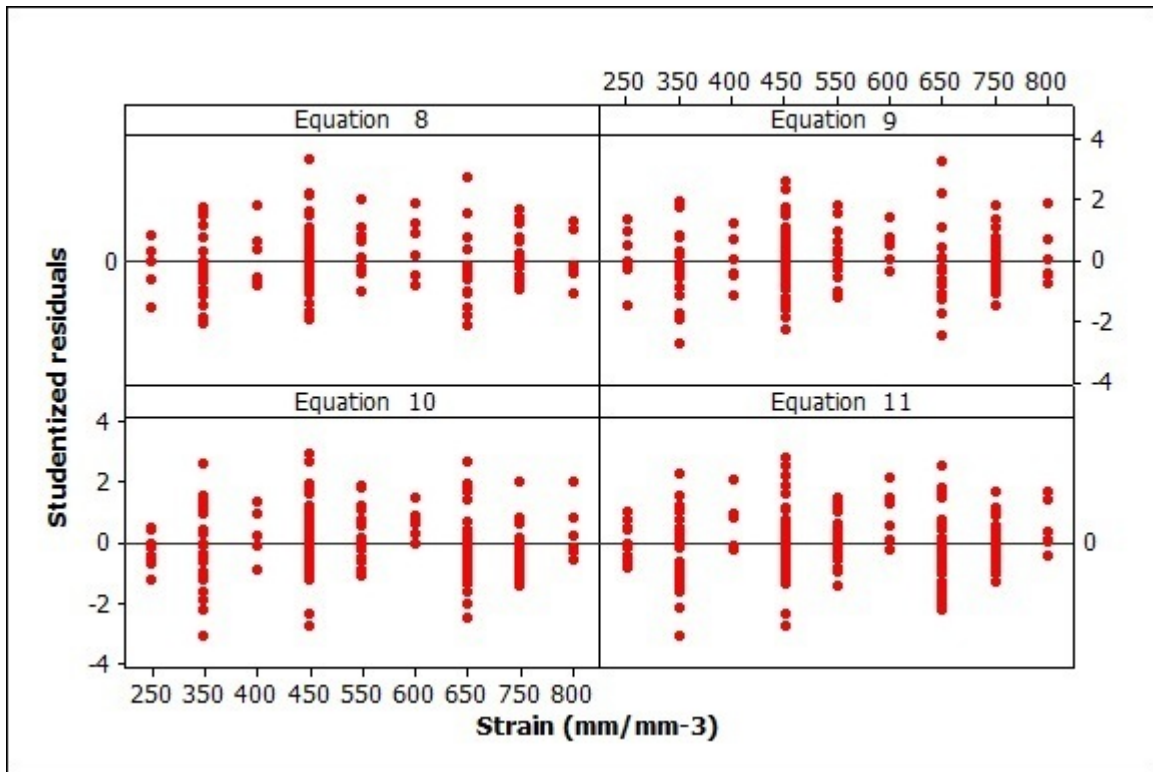
17 Finally, Equation 11 includes tensile strength to characterize the mixture. This model has  
 18 the least favorable regression parameters but is still a valid alternative to predict the PV. The  
 19 advantage of this equation is that  $S_t$  is a variable that can be easily measured in the laboratory and  
 20 the procedure is less time-consuming than other tests considered in this study. Since the majority  
 21 of Costa Rican mixtures are designed with the Marshall method, this option can be easily adapted  
 22 to evaluate fatigue cracking susceptibility with the available resources.

23 As previously mentioned, the main advantage of the dissipated energy approach for  
 24 evaluating fatigue cracking potential in asphalt mixtures is the unique relationship between the  
 25 parameter PV and the number of cycles to failure. The results obtained tend to be more uniform  
 26 because the relationship is not dependent on the mixture properties or test conditions. Figure 9  
 27 shows measured versus fitted PV values for the models developed in Table 2. In all cases, the  
 28 average fitted values fall along the line of equality, with the main difference between the models  
 29 being the amount of data scatter.  
 30



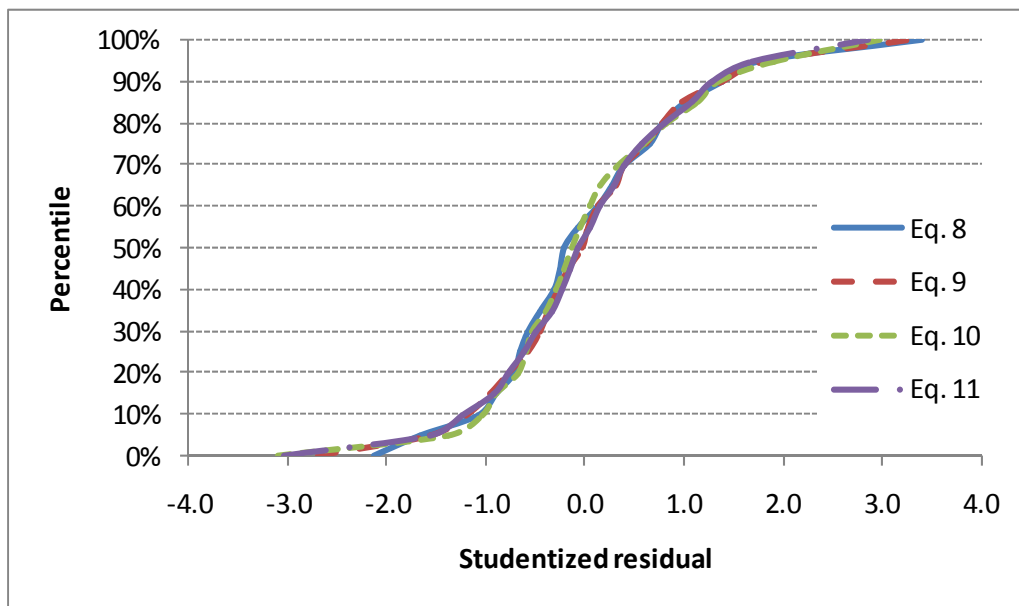
**FIGURE 9** Comparison between measured and calculated PV results for a) Equation 8, b) Equation 9, c) Equation 10 and d) Equation 11.

Figure 10 presents the studentized residuals for every model versus strain level. The plots show that the behavior is very similar for all models. Compared to the residuals obtained with the phenomenological model, values are closer to zero, which is an indication of reduced error. In addition, the amount of extreme values also decreased, with all models having over 99% of the studentized residuals within a range of -3 to 3, as shown in Figure 11.



**FIGURE 10** Residuals versus strain for all PV prediction models.

1  
2  
3



**FIGURE 11** Cumulative distribution of studentized residuals.

4  
5  
6  
7  
8

### Application to Pavement Design

9 Writing the PV- $N_f$  relationship from Table 1 in terms of  $N_f$  and substituting PV with any of the  
10 models obtained in Table 2, a new equation can be developed to estimate the fatigue life as a  
11 function of the corresponding variables. For example, if dynamic modulus, mixture volumetric

1 and gradation data are available, a combination of the PV- $N_f$  relationship and Equation 8 would  
 2 result in the fatigue model shown in Equation 12:

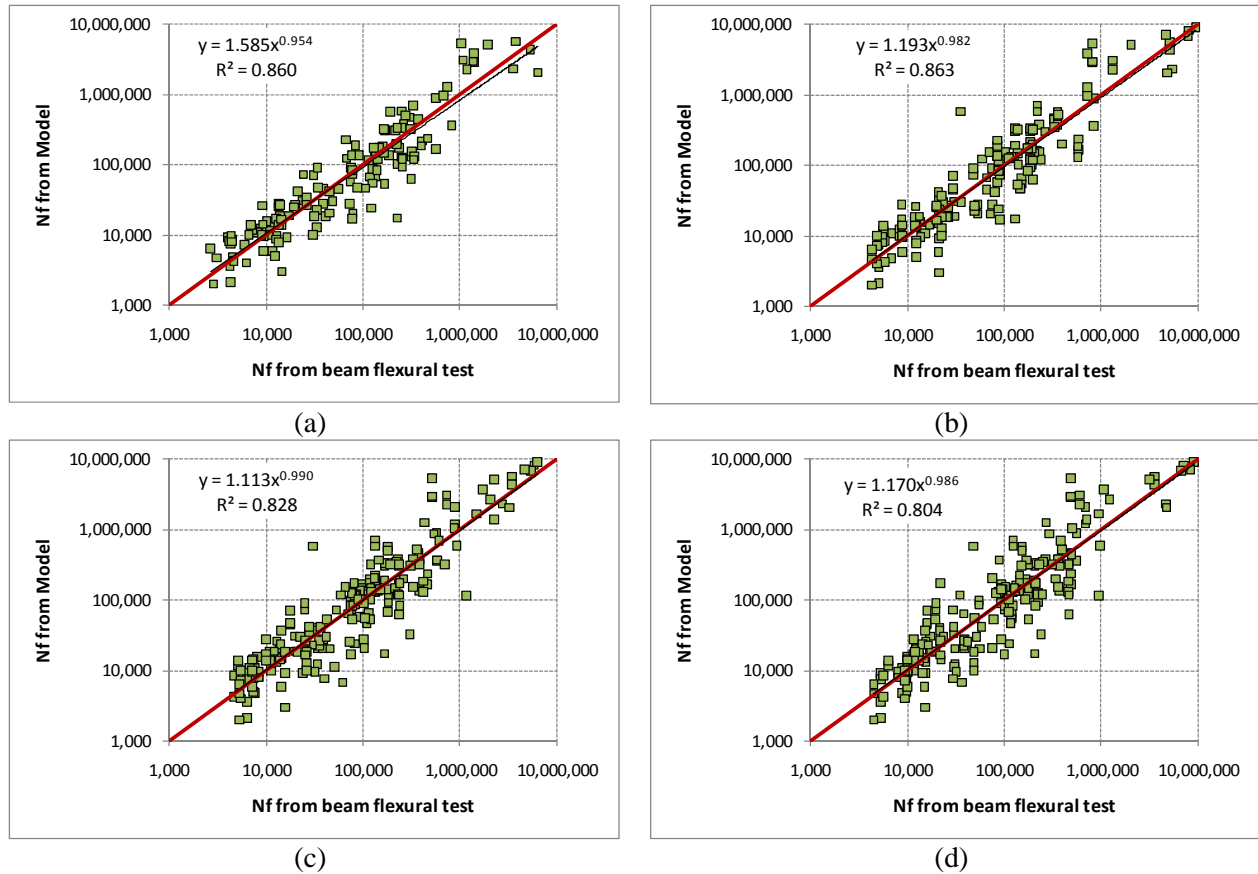
$$N_f = 441.78 \varepsilon^{-5.5838} E^{-4.5664} VP^{-0.7035} GP^{1.1158} \quad (12)$$

4 The number of load applications to failure for a given asphalt mixture will be a function  
 5 of the expected strain level and tensile strength. The same procedure can be applied to estimate  
 6 the fatigue life of asphalt mixtures using the variables in equations 9 to 11, resulting in the  
 7 equations shown in Table 3. Figure 12 shows a comparison between measured and predicted  
 8 fatigue lives for each of these models. As with the PV prediction models, the results generally fall  
 9 along the line of equality and the main difference between the equations is the amount of  
 10 variability observed in the data. This represents an improvement from the results obtained  
 11 through the phenomenological approach, where as previously shown in Figure 3, the best-fit line  
 12 deviated significantly from the line of equality and there was a more data scatter. Compared to  
 13 Figure 3, the coefficients shown in Figure 12 are closer to 1 and the  $R^2$  values are higher (0.80 or  
 14 over), which supports this claim.

15  
 16 **TABLE 3** Fatigue Models for Pavement Design

Eq.	Fatigue Model	Predictor Variables
13	$N_f = 441.78 \varepsilon^{-5.5838} E^{-4.5664} VP^{-0.7035} GP^{1.1158}$	Tensile strain, dynamic modulus, volumetric parameter, gradation parameter
14	$N_f = 2.60 \times 10^{-8} \varepsilon^{-5.3478} E^{-2.2197} \phi^{2.6036}$	Tensile strain, dynamic modulus, phase angle
15	$N_f = 2.94 \times 10^{-9} \varepsilon^{-5.4325} M_R^{-1.2134}$	Tensile strain, resilient modulus
16	$N_f = 3.28 \times 10^{-9} \varepsilon^{-5.5749} S_t^{-1.6557}$	Tensile strain, tensile strength

17  
 18



1 **FIGURE 12** Comparison between measured and calculated fatigue life results for a) Equation  
 2 13, b) Equation 14, c) Equation 15 and d) Equation 16.  
 3

4 It should be noted that the models obtained in Table 3, and any other model derived  
 5 through a similar procedure, are based on laboratory measured fatigue cracking resistance and  
 6 need to be calibrated with field data in order to be incorporated in the pavement design method.  
 7 However, the basic procedure established in this study is a useful alternative to estimate the  
 8 fatigue life.  
 9

## 10 CONCLUSIONS

11  
 12 This study evaluated the dissipated energy approach as an alternative to assess the fatigue life of  
 13 Costa Rican asphalt mixtures. The following conclusions can be drawn:  
 14

- 15 • The concept of a unique relationship between the plateau value (PV) and the number of  
 16 cycles to failure was validated, which included asphalt mixtures typically used in Costa Rica  
 17 as well as other mixture types not widely used, but produced with local materials.
- 18 • The PV parameter can be expressed as a function of response to load and mixture properties.  
 19 Several models were fitted with  $R^2$  values above 0.80. One of the models has the advantage  
 20 of using tensile strength as a measure of mixture stiffness, a result that can be easily obtained.
- 21 • From the equations obtained, it was possible to derive fatigue models to estimate the number  
 22 of cycles to failure. These models can be incorporated into pavement design following  
 23 appropriate calibration with field data.

1           The results from this study are based on laboratory fatigue tests conducted at 20°C. It is  
2 recommended that additional testing be performed at different temperatures to include the  
3 expected range of service temperatures. Tests should include beam flexural fatigue as well as  
4 dynamic modulus, resilient modulus and tensile strength. Additionally, full-scale testing is  
5 required for adequate application of the fatigue models for pavement design.

## 6 7 **REFERENCES**

- 8
- 9 1. Roberts, F.L., P.S. Kandhal, E.R. Brown, D.Y. Lee, and T.W. Kennedy. *Hot Mix Asphalt*  
10 *Materials, Mixture Design, and Construction*. NAPA Education Foundation, Lanham, MD,  
11 Third Edition, 2009.
- 12 2. SHRP, A-404. *Fatigue Response of Asphalt-Aggregate Mixes*. Strategic Highway Research  
13 Program, National Research Council, 1994.
- 14 3. Tayebali, A.A., J.A. Deacon, J.S. Coplantz and C.L. Monismith. Modeling Fatigue Response  
15 of Asphalt Aggregate Mixtures. *Proceedings of Association of Asphalt Paving Technologists*,  
16 Vol. 62, 1993, pp. 385-421.
- 17 4. Pell, P.S. Fatigue of Asphalt Pavement Mixes. *Proceedings of the Second International*  
18 *Conference on the Structural Design of Asphalt Pavements*, Ann Arbor, MI, pp. 577-593.
- 19 5. Diefenderfer, S. *Investigation of Fatigue Properties of Superpave HMA at the Virginia Smart*  
20 *Road*. Ph.D. dissertation. Virginia Polytechnic Institute and State University, October, 2009.
- 21 6. Shen, S. and S. Carpenter. *Dissipated Energy Concepts for HMA Performance: Fatigue and*  
22 *Healing*. Dept. of Civil and Environmental Engineering, Univ. of Illinois at Urbana-  
23 Champaign, Advanced Transportation Research and Engineering Laboratory, ATREL, 2007.
- 24 7. Carpenter, S.H., K.A. Ghuzlan, and S. Shen. A Fatigue Endurance Limit for Highway and  
25 Airport Pavement. In *Transportation Research Record: Journal of the Transportation*  
26 *Research Board*, No. 1832, Transportation Research Board of the National Academies,  
27 Washington, D.C., 2003, pp. 131-138.
- 28 8. Prowell, B.D., E.R. Brown, R.M. Anderson, J.S. Daniel, A.K. Swamy, H. Von Quintus, S.  
29 Shen, S.H. Carpenter, S. Bhattacharjee and S. Maghsoodloo. *Validating the Fatigue*  
30 *Endurance Limit for Hot Mix Asphalt*. NCHRP Report 646. Transportation Research Board of  
31 the National Academies, Washington, D.C., 2010.
- 32 9. Shen, S. and S. Carpenter. Energy-Derived, Damage-Based Failure Criterion for Fatigue  
33 Testing. In *Transportation Research Record: Journal of the Transportation Research Board*,  
34 No. 1723, Transportation Research Board of the National Academies, Washington, D.C.,  
35 2000, pp. 141-149.
- 36 10. AASHTO T 321. *Standard Method of Test for Determining the Fatigue Life of Compacted*  
37 *Hot Mix Asphalt (HMA) Subjected to Repeated Flexural Bending*. American Association of  
38 State Highway Transportation Officials, 2007.
- 39 11. Monismith, C.L., J.A. Epps and F.N. Finn. Improved Asphalt Mix Design. *Proceedings of*  
40 *Association of Asphalt Paving Technologists*, Vol. 54, 1985, pp. 347-406.