Accounting for Censoring and Unobserved Heterogeneity in Pavement Cracking

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Abstract:*Most fatigue cracking models in use have been developed using the Ordinary Least Squares (OLS) method. However, the fatigue cracking data (or any type of cracking data) consists of censored data since it has a lower limit of zero. This can cause bias in the fatigue cracking model, because the data is not continuous but has positive probability mass at zero. Additionally, when data is selected only from pavements that exhibit cracking, bias will result because the estimates are based on a non-random sample. Moreover, bias can also be generated by unobserved factors not included in the fatigue cracking model. This type of bias can be removed by considering the deterioration history of each pavement section, if the unobserved factors are section-specific.*

Based on an LTPP dataset consisting of SPS-1 pavement sections, the authors have modeled fatigue cracking of pavement structures. The data were initially used in modeling fatigue cracking by means of OLS and by a corner solution regression model (tobit) that accounts for the data censoring in fatigue cracking. The tobit model was used, analyzing the data as pooled and also as a panel dataset (by random-effects), to check for possible bias in the model due to unobserved heterogeneity.

The OLS fatigue cracking model exhibits several types of biases due to heterogeneity and erroneous assumptions in the modeling process. The model estimates and test statistics used to evaluate them indicated that the preferred fatigue cracking model was the random effects tobit model because it accounts for the censoring and heterogeneity bias. Estimating the model by accounting for these types of bias in the data resulted in significant changes in the effects of different parameters affecting fatigue through time.

Introduction

In current pavement design and analysis practice, both empirical and mechanistic-empirical types of models are used to characterize the performance of pavement structures. This is because it is not currently possible to characterize the behavior of pavement structures and materials in a completely mechanistic manner, due to the complexity associated in the processes that affect the different pavement layers and the variability associated with the parameters involved.

Consequently, there are many available pavement deterioration models associated with design and analysis procedures throughout the United States, and worldwide. Each of these models has been developed with the goal of characterizing pavement structures under specified conditions. Therefore, it is of the utmost importance to ensure that the models that are currently used, and the ones that will be developed in the future, are calibrated for local conditions and account for the different types of bias and censoring that may be associated with data used in each one of these models.

Flexible pavements can exhibit several types of distress. Many types of distress are due to poor construction, inadequate maintenance, and poor materials, and as such, the distress is not directly related to design. However, there are several distress types that can be mitigated through adequate design of the pavement structure. Examples of such distress types include: (i) rutting, which usually develops in the initial years of the pavement's life and then gradually decreases to lower levels; (ii) fatigue, or alligator cracking that starts to develop during the service life of the pavement with repeated traffic loadings; and (iii) low temperature cracking that can occur at any time during the pavement's service life, due to large temperature differentials.

The current paper focuses on fatigue cracking. In particular, the phenomenon of fatigue crack progression in flexible pavements is addressed. Pavement fatigue cracking is a distress mechanism that occurs mainly in areas subjected to repeated traffic loading (e.g. vehicle wheel path) due to failure of the surface or base layer that is subjected to repeated tensile stresses. Fatigue cracking initiates at the bottom of the asphalt surface (or stabilized base) where the tensile stress or strain due to the applied traffic load is maximum and therefore promote accumulation of damage that will eventually result in the formation of a crack. More simply stated, fatigue cracking is damage occurring in the asphalt mixture due to the repeated or cyclic loading associated to traffic (Adhikari et al., 2009).

Once the crack develops, continued traffic loading, along with other factors, will promote propagation of the crack to the pavement surface which can then be observed as a longitudinal crack (in the direction of traffic). With additional traffic loading, and the combined action of the environment, material properties, and pavement structure, interconnected cracks start to develop (Zhou et al., 2007). From Miller and Bellinger (2003), fatigue cracking "begins as a series of interconnected cracks in early stages, and develops into many-sided, sharp-angled pieces (usually less than 0.3 m on the longest side) characteristically with an alligator pattern in later stages." Based on the Distress Identification Manual for the Long Term Pavement Performance (LTPP) Program (2003), fatigue cracking can be classified into three severity levels: i) Low – the cracked area has none to few interconnected cracks (no spalling and no sealed cracks) and pumping is not evident, ii) Moderate – the cracked area shows interconnected cracks forming a pattern with no signs of pumping (slight spalling and cracks may be sealed), and iii) High – the cracked area shows significantly spalled interconnected cracks in obvious pattern with signs of pumping and movement due to traffic (cracks might be sealed).

Fatigue cracking is a major distress type in flexible pavement structures and is dominant under intermediate temperatures (Wen and Bahia, 2009). Accordingly, proper modeling of this distress type needs to be ensured so that the design predictions are achieved within a small acceptable confidence interval. However, this is challenging because many of the input parameters that are needed for the cracking analysis are difficult to obtain and because the fatigue cracking mechanism is still not completely understood (Molenaar, 2007).

Finally, proper predictions of remaining service life are fundamental in developing an adequate pavement management plan (Kutay et al., 2009). Furthermore, even though fatigue cracking is a structural type of failure, it will generally be associated with a loss in serviceability (deterioration of ride score or increase in roughness) which will also warrant maintenance or rehabilitation activities to improve the ride quality for the road users.

The remainder of this paper is organized as follows: 1) a literature review, focusing on modeling fatigue cracking;2) a discussion of the econometric framework developed to model crack progression in the present study; 3) the details associated to data preparation and its characteristics; 4) a discussion on the empirical results of the estimation, and 5) the paper conclusions.

Modeling of Fatigue Cracking

Previous research in pavement fatigue cracking can be classified into two main groups or categories: i) mechanistic-empirical models, and ii) purely empirical models. In general, both of this model types are empirical, since they are based on statistical models that are applied, under specific conditions, to properly predict or capture fatigue cracking. The main difference between these two types of models is that the purely empirical models are based solely on material, structural, traffic, and environmental properties, while the mechanistic-empirical models incorporate some type of pavement response as an input variable in the estimation of damage or cracking.

Most of the mechanistic-empirical models follow some modification of the universal fatigue law (Finn, 1973; Sun et al., 2003):

$$N_f = k_1 \epsilon^{-k_2} E^{-k_3} \quad (1)$$

where ϵcorresponds to maximum tensile strain at bottom of asphalt layer, E corresponds to the resilient modulus (or a measure of stiffness of the asphalt mixture), k_i are estimated parameters, and N_f corresponds to the total number of cycles to failure due to fatigue cracking (typically defined as a function of the total pavement surface area). Additionally, in many cases the coefficients $k_2$and k_3 are calibrated in the laboratory (Tseng and Lytton, 1990).

In the case of purely empirical models, more sophisticated econometric techniques can be applied to account for the different types of data and outputs that are generally used in developing the models. The fatigue cracking deterioration process is composed of two main components, (i) initiation and (ii) progression, and as such several empirical models have been developed to model these processes. Although some studies have looked at the process jointly (Madanat et al., 1995; Madanat and Shin, 1998), most of the previous efforts have modeled these components separately (Markow and Brademeyer, 1981; Queiroz, 1981; Hodges et al., 1975; Parsley and Robinson, 1982).

More recently, the crack initiation is modeled as a hazard duration model (Paterson, 1987; Shin and Madanat, 2003; Loizos and Karlaftis, 2005; Guler and Madanat, 2011). The hazard function is aimed at capturing the instantaneous probability of failure due to cracking of a pavement structure and consequently is dependent on the definition of failure that is used. The survival and failure functions (cumulative density function associated with failure) can be easily obtained from the hazard function.

Shin and Madanat (2003) used the AASHO Road Test data to fit a Weibull hazard model where cracking was characterized in terms of traffic loading:

$$h(t, \boldsymbol{X}, \boldsymbol{\beta}) = e^{-\gamma \boldsymbol{X\beta}} \gamma t^{\gamma - 1} (2)$$

where γ is a positive parameter associated with the distribution, $\boldsymbol{\beta}$ are estimated parameters to capture the effect of the independent variables $\boldsymbol{X}$ (thickness of surfacing, base and sub base, nominal axle load, and single or tandem axle), and t represents time to crack initiation. Guler and Madanat (2011) also developed a modification of the previous model by including the load of the truck single axle and the tandem axle as dependent variables in the model.Loizos and Karlaftis (2005) similarly fit several hazard models assuming Weibull, Exponential, Lognormal, and Logistic distributions.

A model with fewer restrictions was proposed byNakat and Madanat (2008) were the failure rates were fit using semi-parametric assumptions. The non-parametric component is not limited to any specific distribution assumption. The hazard function that was used is known as the Cox hazard function.Reger et al. (2013)also fit a semi-parametric survival model where field and experimental data were combined to properly account for different factors affecting crack initiation.

Based on the survival analysis approach, time to reach a given failure criteria could also be developed. However the modelsare limited to predicting the time required to reach a certain pavement condition and not the change in deterioration with time or traffic, which can be very useful for pavement design, analysis and management.

Crack progression is typically modeled as a continuous variable using a statistical tool such as Ordinary Least Squares (OLS). In general, these methods are subject to selection bias introduced due to the use of non-random sample of pavements for modeling crack progression (Madanat et al., 1995). This is due to the fact that only pavement sections that present cracking are used in fitting the models. Moreover, it is important to note that the unobserved factors that influence the crack initiation for a particular section are likely to influence the crack progression, since both processes are caused by stresses and strains in the pavement structure associated with traffic loads, given specific environment, material and structural properties. In other words, it is important to allow for the presence of common unobserved factors in modeling crack initiation and crack progression.

In order to account for truncation in the dataset, Madanat et al. (1995) proposed a joint discrete-continuous model to address the issues identified earlier using the dataset from World Bank's road deterioration studies carried out in Brazil from 1975 to 1982. In their framework, (i) a discrete model is used to predict whether the section is cracked or not, and (ii) a regression model is developed for predicting the crack area conditional on the segment being cracked. The model can be expressed as follows:

$$Z_i^* = \boldsymbol{W}_i \boldsymbol{\gamma} + u_i$$

$$Z_i = \begin{cases} 0 & if Z_i^* > 0 \\ 1 & if Z_i^* \leq 0 \end{cases}$$

$$Y_i = \begin{cases} \boldsymbol{X}_i \boldsymbol{\beta} + \epsilon_i if Z_i^* > 0 \\ \quad - if Z_i^* \leq 0 \end{cases} \quad (3)$$

where Z_i^* is a latent variable representing the propensity to distress initiation of pavement section i, Z_i is an indicator variable ($Z_i = 1$ means that cracking has initiated),Y_i corresponds to the cracked area, $\boldsymbol{W}_i$ and $\boldsymbol{X}_i$ are sets of independent variables that are used to characterize cracking, γ and β are the parameters associated with the previous independent variables, and u_i and ϵ_i correspond to random error terms associated with the models. The previous model can be fit using Heckman's 2-step procedure (sequential estimation of the 2 models) or directly by means of Maximum Likelihood Estimation (MLE) if additional assumptions are made.

Using the same dataset, Madanat and Shin (1998) incorporated a random effects term in Eq. (3) to account for unobserved heterogeneity of the data. Unobserved heterogeneity corresponds to unobserved factors that are specific to each pavement section, but might be different between pavement sections. Unobserved heterogeneity can be accounted for by using panel datasets (several observations through time for each pavement section). However, the developed model still has some limitations. Because of the difficulty of performing a MLE estimation with panel data, the joint model that was developed was estimated sequentially rather than simultaneously (using a 2-step procedure) and as demonstrated by Madanat et al. (1995) this results in a loss in efficiency of the parameter estimates. Furthermore, few parameters are used to characterize cracking (structural number and number of wheel passes). Another important aspect not considered in the Madanat and Shin study is the potential presence of unobserved attributes influencing the individual models (crack initiation and progression). For instance, in many pavement databases it is very difficult to clearly determine the quality of construction and materials used. These unobserved attributes potentially influence the likelihood of cracking or crack progression. Ignoring the moderating effect of such unobserved variables can, and in general will, result in inconsistent estimates in nonlinear models (Bhat, 2001; Chamberlain, 1980).

More recently, Madanat et al. (2010) also tried to account for truncation bias, unobserved heterogeneity, and endogeneity bias (correlation between one or more of the independent variables and the error term). Since the purpose was to show the effect of the previous types of bias, the biases are accounted for by means of separate models to address each type individually, following techniques similar to those mentioned previously. The models are fit using Washington DOT PMIS data associated with overlay projects. However, no model that incorporates all these features jointly is presented. Such a model would be of great value since it would ensure that the cracking estimates are unbiased, and more importantly consistent and efficient. The use of a dynamic type model (the change in cracking is a function of cracking during the previous period) as follows was proposed:

$$\Delta Y_{it} = \boldsymbol{X}_{it}\boldsymbol{\beta} + Y_{i,t-1}\gamma + v_{it} + u_i (4)$$

where $\boldsymbol{X}_{it}$ are independent variables, $Y_{i,t-1}$ is the cracking during the previous year, ΔY_{it} is the change in cracking between the previous and the current year, $\boldsymbol{\beta}$andγ are estimated parameters that capture the effect of $\boldsymbol{X}_{it}$ and $Y_{i,t-1}$, and $v_{it} + u_i$ correspond to unobserved factors or model error. However, it has to be noted that $Y_{i,t-1}$ is not necessarily an exogenous variable since it is correlated with the error termu_i. Consequently, use of OLS or panel data models without accounting for the previous correlation will result in biased estimates.

Dong et al. (2013) employed parametric Negative Binomial and zero-inflated Negative Binomial models to simulate the initiation and progression of cracking. The models were estimated by MLE. However, because the data is pooled in order to estimate the models, the unobserved heterogeneity is not being accounted for. Additionally, models such as the Poisson and Negative Binomial regression cannot be readily applied towards fatigue cracking since they are used to predict count data. These types of models are limited to estimating the number of transverse or longitudinal cracks.

The model developed in this paper follows along the lines of Madanat and Shin (1998) and Madanat et al. (2010). The model is intended to simultaneously address unobserved heterogeneity and censoring bias. At the same time it addresses the limitations highlighted above: i) the econometric model is developed to simultaneously model and estimate crack initiation and crack progression, ii) the model accounts for the effect of common unobserved attributes on the initiation and progression components, and iii) the model analyzes the influence of unobserved attributes in the individual components.

Model Definition

The following section introduces the model that is fit as part of this paper. The model that is developed corresponds to a class of models that is generally referred to as corner solution regression models. Corner solution regression models are used when the observed data is in general continuous, but has positive probability mass atone or more specific points (Wooldridge, 2010). In the case of a cracking model, the positive probability mass accounts for the probability that a given pavement section has not initiated cracking, prior to any cracking propagation process. Similar considerations have been applied by Madanat et al. (2002) for the calibration of the AASHO design equations.

The interest of a cracking model (or set of models) should be to determine whether a pavement section has cracked, and if so, what is the amount of cracking that is observed. Therefore, mathematically the interest is in estimating $P(Y=0|\boldsymbol{X})$ and $E(Y|\boldsymbol{X})$, where Y in this case corresponds to a measure of cracking, $\boldsymbol{X}$corresponds to independent variables that are used to characterize cracking, $P(\cdot)$ corresponds to the probability that there is no cracking, and $E(\cdot)$ corresponds to the expected amount of cracking for a given set of conditions $\boldsymbol{X}$.

Because of the $Y=0$ corner solution, the typical cracking progression model that is defined as $E(Y|\boldsymbol{X}) = \boldsymbol{X}\boldsymbol{\beta}$(typical OLS model), where $\boldsymbol{\beta}$ corresponds to the model parameters to be estimated, is biased because necessarily$Y \geq 0$, and therefore the expected value of Y cannot be linear. Otherwise, there could be combinations of $\boldsymbol{X}$ and $\boldsymbol{\beta}$ that would result in cracking values that are negative.

In order to address the previous shortcomings, we can define the general structural form of the cracking statistical model as follows,

$$\begin{aligned} Y_{it}^* &= \boldsymbol{X}_{it}\boldsymbol{\beta} + u_{it}, \qquad t = 1,2,\dots,T \\ Y_{it} &= \max(0, Y_{it}^*) \end{aligned} \tag{5}$$

where Y_{it}^* is a latent variable or artificial construct to indicate the amount of cracking, which in reality is given by Y_{it}. The "i" indicates a specific pavement section, and the "t" represents a given time observation for each "i" section. $\boldsymbol{X}_{it}$ and $\boldsymbol{\beta}$ are as previously defined and u_i are unobserved factors that are not explicitly included in $\boldsymbol{X}_{it}$ but that have an effect on Y_{it}^*. It has been assumed that $u_{it} \sim Normal(0, \sigma^2)$. This type of model can be referred to as a Type I Tobit model (Amemiya, 1984). The model also accounts for the fact that there is heteroskedasticity in the variance associated with Y_{it}: $V(Y_{it}|\boldsymbol{X}_{it})$. Heteroskedasticity means that the variance changes with $\boldsymbol{X}_{it}$ as is typically the case with pavement performance.

Based on (5), the quantities of interest, $P(Y = 0|\boldsymbol{X})$ and $E(Y|\boldsymbol{X})$, can be defined as follows,

$$P(Y_{it} = 0|\boldsymbol{X}_{it}) = 1 - \Phi(\boldsymbol{X}_{it}\boldsymbol{\beta}/\sigma)$$
$$E(Y_{it}|\boldsymbol{X}_{it}) = \Phi(\boldsymbol{X}_{it}\boldsymbol{\beta}/\sigma) \cdot E(Y_{it}|\boldsymbol{X}_{it}, Y_{it} > 0) \quad (6)$$

where $\Phi(\cdot)$ is the standard Normal cumulative density function. Finally, using several properties of the Normal distribution (Wooldridge, 2010) it can be obtained that,

$$E(Y_{it}|\boldsymbol{X}_{it}, Y_{it} > 0) = \boldsymbol{X}_{it}\boldsymbol{\beta} + \sigma\left[\frac{\phi(\boldsymbol{X}_{it}\boldsymbol{\beta}/\sigma)}{\Phi(\boldsymbol{X}_{it}\boldsymbol{\beta}/\sigma)}\right] \quad (7)$$

where $\phi(\cdot)$ is the standard Normal density function. Note that the term on the right hand side of (7) is positive for any combination of $\boldsymbol{X}_{it}$ and $\boldsymbol{\beta}$.

<u>Unobserved Effects</u>

Up to this point we have not addressed unobserved effects in (5). To account for this section specificunobserved effects, c_i, we can re-specify (5) as,

$$Y_{it} = \max(0, \boldsymbol{X}_{it}\boldsymbol{\beta} + c_i + u_{it}), \qquad t = 1,2,\ldots,T$$
$$u_{it}|\boldsymbol{X}_i, c_i \sim Normal(0, \sigma_u^2) \quad (8)$$

In the previous model, c_i corresponds to the latent effect (heterogeneity or individual effect) and corresponds to behavior strictly associated to group "i" (pavement section "i"). In contrast, u_{it} corresponds to the average random component associated with all Y_{it} (unobserved factors that are shared by all pavement sections).

To avoid imposing the random effects assumption that $\boldsymbol{X}_{it}$ and c_i are strictly uncorrelated, it can be assumed that $c_i|\boldsymbol{X}_{it} \sim Normal(\psi + \overline{\boldsymbol{X}}_i\boldsymbol{\xi}, \sigma_a^2)$, where σ_a^2 is the variance of a_i in $c_i = \psi + \overline{\boldsymbol{X}}_i\boldsymbol{\xi} + a_i$. Then the random effects Tobit model can be specified as follows,

$$Y_{it} = \max(0, \psi + \boldsymbol{X}_{it}\boldsymbol{\beta} + \overline{\boldsymbol{X}}_i\boldsymbol{\xi} + a_i + u_{it}), \qquad t = 1,2,\ldots,T$$
$$u_{it}|\boldsymbol{X}_i, a_i \sim Normal(0, \sigma_u^2)$$
$$a_i|\boldsymbol{X}_i \sim Normal(0, \sigma_a^2) \quad (9)$$

Finally, based on the random sample$\{(\boldsymbol{X}_{it}, Y_{it}): i = 1,2,\ldots,N; t = 1,2,\ldots,T\}$, the ψ, $\boldsymbol{\beta}$, $\boldsymbol{\xi}$, σ_a^2, and σ_u^2 parameters can be estimated by means of maximum likelihood estimation by maximizing the following likelihood function,

$$\ell = \int_{-\infty}^{\infty}\left[\prod_{i,t}\{1 - \Phi((\psi + \boldsymbol{X}_{it}\boldsymbol{\beta} + \overline{\boldsymbol{X}}_i\boldsymbol{\xi})/\sigma_u)\}^{1[Y_{it}=0]}\{(1/\sigma_u)\phi((Y_{it} - \psi - \boldsymbol{X}_{it}\boldsymbol{\beta} - \overline{\boldsymbol{X}}_i\boldsymbol{\xi})/\sigma_u)\}^{1[Y_{it}>0]}\right]\phi(a_i)da_i \quad (10)$$

As part of the present study, the MLE estimates of the parameters were obtained by solving the previous function based on Ox programming language and using adaptive quadrature to approximate (10).

Case Study

The proposed models were estimated using LTPP data. The LTPP experiment, which began in 1989, contains data on pavement constructions, materials, traffic and performance. The study is composed of a number of experiments monitored at multiple locations across North America.

For the purpose of this paper, 116 sections throughout 18 States of the United States we selected. The selected sections are part of the Specific Pavement Studies (SPS) experiments SPS-1 (Strategic study of structural factors for flexible pavements). The data were extracted from the LTPP Standard Data Release 25.0 (January 2011). This is consistent with the data used to estimate the deterioration models included in the Mechanistic Empirical Pavement Design Guide (MEPDG), but the data have been significantly updated to reflect the latest data available. The geographic distribution of the selected sections is shown in Figure 1;however note that at each location several LTPP sections were constructed. SPS-1 sections were selected because they have been monitored on the LTPP experiment since the moment of their initial construction during the early 1990s and as such no assumption needs to be made as to the initial condition of the pavement structure.

<FIGURE 1>

Out of all the existing SPS-1 experiment sections, the pavement sections included in the current study were selected based on the availability of the pavement section specific information that was identified by previous research studies as having an important effect on fatigue cracking, and some additional information that was considered by the authors to be relevant in modeling this distress type on pavement structures.

The cracking information was obtained from the Monitoring Module of the LTPP database, specifically the fatigue cracking records. Inspections performed between 1993 and 2004 were obtained for the selected sections. A query of all the selected pavement sections produced a total of 349 individual pavement records. Therefore, the dataset consists of an unbalanced panel where on average there are approximately 3.01 observations per pavement section.

Data Description

The data used in the current study was obtained from the LTPP database. A description of the significant explanatory variables, and dependent variable, that were used in estimating the models follow:

- $Crack_{it}$: Area of fatigue cracking in pavement section "i" at time "t", where $t =$ number of years since the pavement section was initially built (and no maintenance or rehabilitation has been performed during the time "t"). By definition, crack initiation occurs at time $t = t_{c0}$, where t_{c0} corresponds to the time after initial construction when cracking is firstdetected on the pavement surface.
- T_{AC_i}: Total thickness of the asphalt concrete layers in pavement section "i" in inches.
- T_{B_i}: Total thickness of the base layers in pavement section "i" in inches.
- $Treat_{B_i}$: Is a dummy variable which indicates whether the base layers have been treated by any means (1 indicates treatment of base layer, 0 otherwise).
- AC_i: Percentage of asphalt binder content on bottom asphalt concrete layer in pavement section "i".
- Va_i: Percentage of air voids on bottom asphalt concrete layer in pavement section "i".
- $AADTT_{it}$: Average Annual Daily Truck Traffic during year t on pavement section "i".
- $Precip_{it}$: Total annual precipitation during year "t" on pavement section "i", in millimeters.
- $Snow_{it}$: Total annual snowfall during year "t" on pavement section "i", in millimeters.
- $Days32_{it}$: Total number of days during year "t" when the temperature on pavement section "i" was above 32°C.
- $Days0_{it}$: Total number of days during year "t" when the temperature on pavement section "i" was below 0°C.

Other variables were also evaluated, but were dropped as explanatory variablesbecause of low statistical significance and high correlation with some of the previously reported explanatory variables, for all estimated models. Some of these include the maximum annual average temperature, minimum annual average temperature, and the freeze index.

Fatigue Cracking Model Estimation Results

The fatigue cracking model parameters were initially estimated using OLS (same methodology followed by several research studies where censoring of the data is not considered), and by pooled tobit (corner solution regression model) while pooling the dataset. The pooling of the data means that each data observation is considered as a separate pavement section and the unobserved heterogeneity within pavement sections is not accounted for. Additionally, the fatigue cracking model was estimated taking advantage of the panel dataset that is being used by means of a random effects Tobit approach, while accounting for endogeneity due to factors that are specific to each pavement section, but do not change over time.

The mathematical formulation for the estimated and analyzed models is the following:

OLS: $Crack_{it} = \boldsymbol{X}_{it}\boldsymbol{\beta} + u_{it}, \qquad t = 1,2,\dots,T$ (11)

Pooled Tobit: $Crack_{it} = \max(0, \boldsymbol{X}_{it}\boldsymbol{\beta} + u_{it}), \qquad t = 1,2,\dots,T$ (12)

Random Effects Tobit: $Crack_{it} = \max(0, \boldsymbol{X}_{it}\boldsymbol{\beta} + \overline{\boldsymbol{X}}_i\boldsymbol{\xi} + a_i + u_{it}), \qquad t = 1,2,\dots,T$ (13)

where $\boldsymbol{X}_{it} = [1, t, T_{AC}, T_B, Treat_B, AC, Va, AADTT, Precip, Snow, Days32, Days0\ ]$, $\overline{\boldsymbol{X}}_i$ corresponds to the average values of $\boldsymbol{X}_{it}$ for each i, and $\boldsymbol{\beta}, \boldsymbol{\xi}, a_i, u_{it}$are the parameters to be estimated**.**

The parameter estimates (and associated standard errors shown in parenthesis with italics font), the asymptotic t-statistics, and the associated p-values for the regressions with the pooled dataset are shown in Table 1, while the random effects Tobit regression estimates and their associated t-statistics and p-values are given in Table 2. The variance estimates, as well as model fit parameters, for the three different models are shown in Table 3.

<FIGURE 2>

<TABLE 1>

<TABLE 2>

<TABLE 3>

For the purpose of comparison, the fit of the models is shown on Figure 2. From the figure, it is clear how the predicting power is increased from the predictions as unobserved heterogeneity is considered in the estimation procedure. This also functions as an indication that the bias from the OLS model and pooled Tobit model is being reduced. Visually, the pooled Tobit approach does not seem to improve on the fit of the model as compared to the OLS model. However, note how the OLS model allows for fatigue cracking predictions to be less than 0. This has to do with the fact that the OLS model completely ignores how the data is left censored at 0 (corner solution). On the other hand, this phenomenon cannot be observed with the pooled Tobit model since it accounts for the data censoring. It is important to emphasize that, in Figure 2, the randomeffect has not been included in the prediction for comparison purpose with the other models. If the random effects were to be included, the fit of the data on the graph would improve considerably, but, as highlighted by Chu and Durango-Cohen (2008), inclusion of these section specific terms limits the use of the model to the specific conditions (ie. materials, environment) that were observed in the original dataset. However, even if the random effects terms are not included, the model will still represent the average population behavior and is consistent since the unobserved heterogeneity was considered during the estimation procedure.

In general, the pooled Tobit regression provides a slightly better fit to the data, as compared to the OLS model, when the data are pooled together. This is measured by higher F-test statistics for the joint test that all the model parameters are significantly different from zero (13.06 vs. 10.58) and better correlation between the observed data and the estimated predictions, R (0.76 vs. 0.51). However, also note that the standard error associated with the OLS models are smaller, generally by an order of 3, than those obtained by the pooled Tobit estimation.

Also interesting to note are the differences in the model parameters between the two models. A small difference would indicate that the effect of fatigue cracking being a censored variable has small-to-no effect on the predictions. If this were the case, the use of OLS would be appropriate in modeling this type of distress. However, significant differences between the values of the parameter estimates can be observed. This indicates that accounting for data censoring is correct, and that not considering it would produce biased and inconsistent estimates. One of the most important differences between the estimates can be observed for the case of $Treat_B$ (indicates if the base layers have been asphalt treated) and Va (air void content), which the OLS model indicates as having a negative impacts on fatigue cracking, while for the case of the pooled Tobit estimates have the opposite meaning. The other important difference between estimates can be observed on $AADTT$ where OLS estimation indicates that an increase in truck traffic actually reduces fatigue cracking. It is clear that this is incorrect since traffic loading is one of the main factors involved in the fatigue cracking process, as can be observed from the pooled Tobit estimates.

With the exception of the pooled Tobit model, the overall standard deviation for the other models is numerically similar. However, the random effects Tobit model distinguishes between the standard error associated with the population, and the variance associated with each specific pavement section (unobserved section specific attributes that are constant through time and cannot be captured by the pooled data models). This turns out to be a very important component of the model variance since it explains 22.8% of the total model variance. The average model results are shown in Figure 3.

<FIGURE 3>

Consequently, the panel data random effects Tobit model should be used to account for the unobserved heterogeneity that is not captured by the pooled Tobit model and the bias due to censoring that is not accounted for by the OLS model. A t-test was used on the standard deviation associated with the random effects to test the hypothesis that $\sigma_u = 0$, or that the pooled data models are indeed correct and the variability associated with the unobserved heterogeneity is not important. The resulting t-statistic was 3.87, which corresponds to a p-value of less than 10^{-3} (significant at any statistical level of confidence). Therefore, the null hypothesis that there is no variability associated with unobserved heterogeneity within each pavement section can be confidently rejected. This indicates conclusively that the pooled data models are inappropriate for predicting fatigue cracking.

Finally, the normality assumption associated with the unobserved effects u_{it} was also verified. A normal probability plot showing the residuals associated to the random effects Tobit model is shown on Figure 4. The figure indicates that the residuals align properly which is an indicator of normality. The assumption was also verified by means of an Anderson–Darling(AD) normality test which resulted in a value of 1.886, with an associated p-value < 0.005. The AD statistics verifies that the normality assumption is adequate.

<FIGURE 4>

Conclusions

Regardless of whether pavement design and analysis is performed by means of empirical methods or mechanistic empirical methods, the use of empirical transfer functions to characterize deterioration of the pavement structure are fundamental. Consequently, the development of such empirical deterioration models has to be based on statistically sound techniques that account for the type of data, and the properties of the data that are at hand.

Based on the current study, it was clearly evidenced that the use of simple regression techniques such as OLS will result in biased estimates due to the censored nature of the dependent variable (area of fatigue cracking). The censoring, or corner solution concern, results from the fact that fatigue cracking has a lower limit of 0, and as such, negative values cannot be observed and are meaningless. This can be corrected by using a corner solution regression model to account for the nature of the dependent variable, thus removing the bias due to censoring of the data.

The random effects Tobit model developed in the current study also allowed for accounting of unobserved heterogeneity. It was shown that the traditional OLS fatigue cracking model produced estimates that exhibit omitted variable bias because of heterogeneity that is present in the data because of unobserved section-specific variables. However, accounting for omitted variable bias due to unobserved heterogeneity is not only important from a merely statistical standpoint. By comparing the estimates of the OLS model and the random effects Tobit model, considerable differences can be observed in the expected effect of the different factors that have an effect on fatigue cracking.

Based on the random effects Tobit model, time since the pavement section was initially open to traffic, truck traffic, the volumetric properties of the asphalt mix (air void content and asphalt binder content), and the environmental variables have the largest effect on cracking. This conclusion is similar to that obtained from the OLS model estimation, with the difference that the environment has less effect on cracking in the OLS predictions. This similarity is expected due to the importance of these variables on cracking of pavement structures.

Because the pavement fatigue cracking model, as well as any other pavement deterioration or transfer function models are developed empirically, it is imperative that modeling techniques to account for all the complexities of the data available are used. This was the purpose of the random effects Tobit model that was developed in the current study. However, it is important to note that the model is based on a specific sample obtained from the LTPP experiment. Therefore, it is important that the model be calibrated or refit as additional LTPP data becomes available, or based on a completely different dataset, for different conditions and regions, so that the unobserved factors associated with different materials, construction practices, structure types, traffic, and environment be more precisely captured.

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Figure Caption List

Table 1. Estimated Parameters and Statistics for Fatigue Cracking Model (Pooled Data)

Parameter	*OLS*			*Pooled Tobit*		
	Estimates (Std. Err.)	*t-statistic*	*p-value*	*Estimates (Std. Err.)*	*t-statistic*	*p-value*
Intercept	-222.61 (*39.12*)	-5.69	0.00	-605.09 (*117.83*)	-5.14	0.00
t	3.94 (*0.80*)	4.91	0.00	10.73 (*2.45*)	4.38	0.00
T_{AC}	0.28 (*1.09*)	0.26	0.80	1.63 (*3.02*)	0.54	0.59
T_B	-0.21 (*0.57*)	-0.36	0.72	-1.5 (*1.44*)	-1.05	0.30
$Treat_B$	3.08 (*4.92*)	0.63	0.53	-6.22 (*12.58*)	-0.49	0.62
AC	34.93 (*7.62*)	4.58	0.00	84.26 (*25.21*)	3.34	0.00
Va	10.96 (*3.90*)	2.81	0.01	-6.98 (*12.57*)	-0.56	0.58
$AADTT$	-0.01 (*0.01*)	-1.63	0.11	0.1 (*0.03*)	3.00	0.00
$Precip$	-0.02 (*0.01*)	-1.65	0.10	-0.04 (*0.03*)	-1.58	0.12
$Snow$	-0.01 (*0.01*)	-0.54	0.59	-0.04 (*0.03*)	-1.68	0.09
$Days32$	-0.12 (*0.11*)	-1.07	0.28	-0.51 (*0.36*)	-1.43	0.15
$Days0$	0.34 (*0.10*)	3.37	0.00	1.48 (*0.28*)	5.32	0.00

Table 2. Estimated Parameters and Statistics for Fatigue Cracking Model(Considering Unobserved Effects)

Parameter	*Random Effects Tobit*		
	Estimates (Std. Err.)	*t-statistic*	*p-value*
Intercept	525.34 (*256.12*)	2.05	0.04
t	8.99 (*2.85*)	3.16	0.00
T_{AC}	1.01 (*1.87*)	0.54	0.59
T_B	-0.93 (*0.89*)	-1.04	0.30
$Treat_B$	-11.27 (*8.11*)	-1.39	0.17
AC	71.54 (*32.47*)	2.20	0.03
Va	-44.14 (*20.39*)	-2.17	0.03
$AADTT$	-0.11 (*0.04*)	-2.72	0.01
$\overline{AADTT}$ (*)	0.76 (*0.08*)	9.31	0.00
$Precip$	0.11 (*0.04*)	2.58	0.01
$\overline{Precip}$ (*)	-0.81 (*0.11*)	-7.72	0.00
$Snow$	0.05 (*0.02*)	2.03	0.04
$\overline{Snow}$ (*)	-0.33 (*0.06*)	-5.79	0.00
$Days32$	0.44 (*0.32*)	1.36	0.17
$\overline{Days32}$ (*)	-11.15 (*1.67*)	-6.66	0.00
$Days0$	-0.12 (*0.68*)	-0.18	0.86
$\overline{Days0}$ (*)	0.36 (*0.84*)	0.44	0.66

(*) Variables with a bar, $(\bar{X}_i)$, correspond to the mean value of the variable X_{it} throughout the observation period available for pavement section i.

Table 3. Estimates of Variance Components for All the Models

Estimate	*OLS*	*Pooled Tobit*	*Random-Effects Tobit*
σ_a	33.26	62.71	32.14
σ_u	-	-	17.45
$\sigma_w = \sqrt{\sigma_a^2 + \sigma_u^2}$	-	-	36.57
R(**)	0.51	0.76	0.90
F	10.58	13.06	24.39

(**) R defined as the correlation between observed and predicted values from each regression model so that parameter is comparable between models.