# DEVELOPMENT OF AN IMPROVED AND MORE EFFECTIVE DYNAMIC MODULUS E* MODEL FOR MIXTURES IN COSTA RICA BY MEANS OF ARTIFICIAL NEURAL NETWORKS

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**ABSTRACT**

Various dynamic modulus (E*) predictive models have been developed to estimate E* as an alternative to laboratory testing. The most widely used model is the 1999 I-37A Witczak predictive equation based on North American mixtures laboratory results. The differences in material properties, traffic information, and environmental conditions for Latin American countries make it necessary to calibrate these models using local conditions. Consequently, the National Laboratory of Materials and Structural Models at the University of Costa Rica (in Spanish, LanammeUCR) has previously performed a local calibration of this model based on E* values for different types of Costa Rican mixtures. However, further research has shown that there is still room for improvement in the accuracy of the calibrated model (Witczak-Lanamme model) based on advanced regression techniques such as artificial neural networks (ANN).

The objective of this study was to develop an improved and more effective dynamic modulus E* predictive regression model for mixtures in Costa Rica by means of ANN based models. A comparison of the predicted E* values among the Witczak model, Witczak-Lanamme model and the new and improved model based on artificial neural networks (ANN-Lanamme model) indicated that the former not only met the model adequacy checking criteria but also exhibited the best goodness of fit parameters and the lowest overall bias. The findings of this study also supported the use of more advanced regression techniques that can become a more attractive alternative to local calibration of the Witczak I-37A equation.

**INTRODUCTION**

The most important asphalt concrete mixture property influencing the structural response of a flexible pavement is the dynamic modulus (E*). For a specific mixture, temperature, rate of loading and aging significantly influence this property. E* is also the primary hot-mix asphalt (HMA) material property input at all three hierarchical levels in the new mechanistic empirical pavement design guide (MEPDG) (*ARA, 2004*).

Various E* predictive models have been developed to estimate E* as an alternative to laboratory testing. The most widely used model is the 1999 I-37A Witczak predictive model based on conventional multivariate regression analysis of laboratory test data. Because of this, the Witczak model has been evaluated using many different datasets and has been calibrated to several regions. However, use of the model with no calibration for local mixtures should be limited.

A study by the University of Minnesota (on mixtures from four cells at Mn/ROAD) found that the Witczak predictive equation fitted the data relatively well in some locations at intermediate and low temperatures, but for other locations the differences were significant (*Clyne et al., 2003*). This study concluded that the Witczak equation should be used with caution and that further research was needed to validate the Witczak equation for mixtures typically used in Minnesota.

A study at the University of Florida (*Birgisson et al. 2005*), evaluated the Witczak predictive equation for 28 mixtures typical to Florida. Overall, it was found that the Witczak predictive equation resulted in a slight bias for the mixtures investigated. However, the results also allow for a correction of the bias between predicted and measured E* by means of statistical calibration. It was also found that E* predictions at higher temperatures generally were closer to measured values than predictions at lower temperatures, suggesting that the database used to develop the Witczak model could be restricted to predicting the modulus of mixtures tested at higher temperatures, or that, for the mixtures studied, the sigmoidal function used may produce slightly biased E* values at lower temperatures. Finally, it was concluded that when testing results are not available, reliable first order estimates of E* for mixtures typical to Florida can be obtained with the Witczak predictive equation, by applying a correction factor obtained from the testing of local mixtures.

In a study by North Carolina State University (*Kim et al., 2005*), 41 mixtures commonly used in North Carolina were used to evaluate the prediction accuracy of the Witczak model and the influence of some mixture variables in the prediction of E*. The study showed that Witczak's predictions for cooler temperatures were better than at warmer temperatures. This is the opposite of what was observed in Florida and thus highlights the importance of proper calibration.

A study by Schwartz (*2005*) at the University of Maryland evaluated the accuracy and robustness of the Witczak predictive equation through a set of sensitivity and validation analyses, using the same database with which the Witczak model was calibrated, plus an independent set of laboratory E* test data for 26 other mixtures. The validation of the Witczak model against the independent set of data showed an agreement between predicted and measured E* values that was nearly as good as for the calibration data set, but with a slight positive bias (predicted values were generally higher than the measured data) which was higher for lower stiffness/higher temperature conditions.

The University of Arkansas study on 12 different mixtures showed a good correlation between the Witczak predicted E* values and those measured in the laboratory (*Tran and Hall, 2005*). The goodness-of-fit statistics showed that the prediction of E* for the mixtures used in the study ranged from very good to excellent, according to the subjective criteria used. However, the A and VTS parameters used in the Witczak predictive equation were the default values proposed in the MEPDG and not directly calculated.

The Louisiana Transportation Research Center conducted a study on two 25-mm Superpave mixtures with two different binder types to compare two simple performance tests, performed in two laboratories (*Mohammad, 2005*). The prediction capability of the Witczak model was also evaluated. It was also found that the E* can provide consistent results for plant-produced mixtures. Another finding was that the E* was sensitive to different binder contents in the mixture. They concluded that the Witczak model can predict E* values with a reasonable reliability.

Another study developed by Dongré et al. (*2005*) showed that the Witczak model was able to produce reasonable predictions of dynamic modulus when compared to data from mixtures tested in laboratory. However, they also found that both models needed to be corrected or refined to more accurately predict E* values from production samples. Currently, the model under-predicts E* values when higher binder contents or air voids than those indicated by the mix design are used in production samples.

Robbins and Timm (*2011*), evaluated three E* predictive models (Witczak 1-37A, Witczak 1-40D, and Hirsch) with the use of 18 HMA plant-produced, lab-compacted mixtures (representative of general-use mixtures used in the southeastern United States) that were placed at the 2006 National Center for Asphalt Testing Test Track. The Hirsch model for estimating HMA modulus is based on a law of mixtures for composite materials (*Christensen et al., 2003*) which utilizes the shear modulus of the binder, G*, and volumetric properties of the mix to predict E*. E* predictions were made at three temperatures and three frequencies for direct comparison with measured values. The Witczak models had the greatest deviation from measured values, and the Witczak 1-40D model overestimated E* by approximately 61%. The Hirsch model most accurately predicted the moduli for the 2006 Test Track mixtures.

Singh et al. (*2011*) also evaluated the Witczak I-37A model for their use in estimating the dynamic modulus of selected HMA mixtures that are commonly used in Oklahoma. The performance of the predictive model was evaluated by three approaches: goodness-of-fit statistics, comparison of the measured and predicted values, and local bias statistics (slope, intercept, and average error). Analyses of the results showed that the predictive power of the model varied with the temperature and air void levels of a compacted specimen. A calibration factor was developed for the model to obtain an accurate estimate of dynamic modulus. This calibration was considered helpful for Level 2 and Level 3 designs of the Mechanistic–Empirical Pavement Design Guide (MEPDG).

El-Badawy et al. (*2012*), evaluated the influence of the binder characterization input level on the performance of the MEPDG E* predictive models. 27 HMA mixtures commonly used in Idaho were investigated. Results showed that the performance of the investigated models varies based on the temperature and the binder characterization method. The NCHRP 1-37A E* model along with MEPDG level 3 binder inputs yielded the most accurate and least biased E* estimates. The accuracy of this model was further enhanced by introducing a local calibration factor.

In the case of Costa Rica, a similar analysis was performed to ensure that the Witczak model could be readily applied to local mixtures (*Loria et al., 2011*). The Witczak model was identified to produce slightly biased predictions of E* when compared to several gradations, mostly of typical use in the Country. As was the case with some of the previous studies, the model showed positive bias at higher stiffness/lower temperature conditions. Consequently, a calibrated Witczak model was fitted using nonlinear regression.

Far et al. (*2009*), presented the outcomes from a research effort to develop models for estimating the dynamic modulus of hot-mix asphalt (HMA) layers on long-term pavement performance test sections. The goal of their study was to develop a new, rational, and effective set of dynamic modulus E* predictive models. These predictive models used artificial neural networks (ANNs) trained with the same set of parameters used in the modified Witczak and Hirsch models. Modulus values from multiple mixtures and binders were assembled from existing national efforts and from data obtained at North Carolina State University. The results show that the predicted and measured E* values were in close agreement when ANN models were used.

A paper presented by Ceylan et al. (*2009*) discussed the accuracy and robustness of the various predictive models (Witczak I-37A and I-40D and ANN-based models) for estimating E* values. The ANN-based E* models using the same input variables exhibit significantly better overall prediction accuracy, better local accuracy at high and low temperature extremes, less prediction bias, and better balance between temperature and mixture influences than do their ordinary least squares (OLS) regression-based counterparts. As a consequence, the ANN models as a group are better able to rank mixtures in the same order as measured E* for fixed environmental and design traffic conditions.

Singh et al., (*2012*) developed an artificial neural network to predict dynamic modulus of twenty different HMA mixes comprised of various sources, sizes, types of aggregates, and different volumetric properties. The ANN-based model was developed considering the following input variables: aggregate shape parameters (i.e., angularity, texture, form, and sphericity), frequency, asphalt viscosity, and air voids of compacted samples. The shape parameters of different sizes of coarse and fine aggregates were measured using an automated aggregate image measurement system (AIMS). The results showed that the inclusion of aggregate shape parameters can be used as independent parameters in a model for estimating the dynamic modulus of hot mix asphalt.

In summary, the I-37A Witczak predictive model has worked well in some cases and not so well in others. Calibration of this equation has also been implemented by several researchers while others decided to utilize the 2006 Witczak model or decided to adopt a different approach such as the Hirsh model. Finally, the use of more advanced regression techniques has also proven to be a more attractive alternative to calibration of the Witczak I-37A equation.

## OBJECTIVE

The objective of this study was to develop an improved and more effective dynamic modulus *E** predictive regression model for mixtures in Costa Rica by means of artificial neural network (ANN) based models.

## MIXTURE CHARACTERIZATION AND EVALUATION

From 2007, LanammeUCR has conducted a laboratory evaluation of the applicability of the Witczak Model to a typical aggregate source and one type of asphalt binder produced in Costa Rica *(Loria et al., 2011)*. The flow chart presented in Figure 1 summarizes the experimental plan of the study.

### *Aggregate Characterization*

The study involved one aggregate source (from a northeast region of the country called Guápiles). The aggregate is extruded from igneous deposits along a river. The aggregate properties are shown in Table 1.

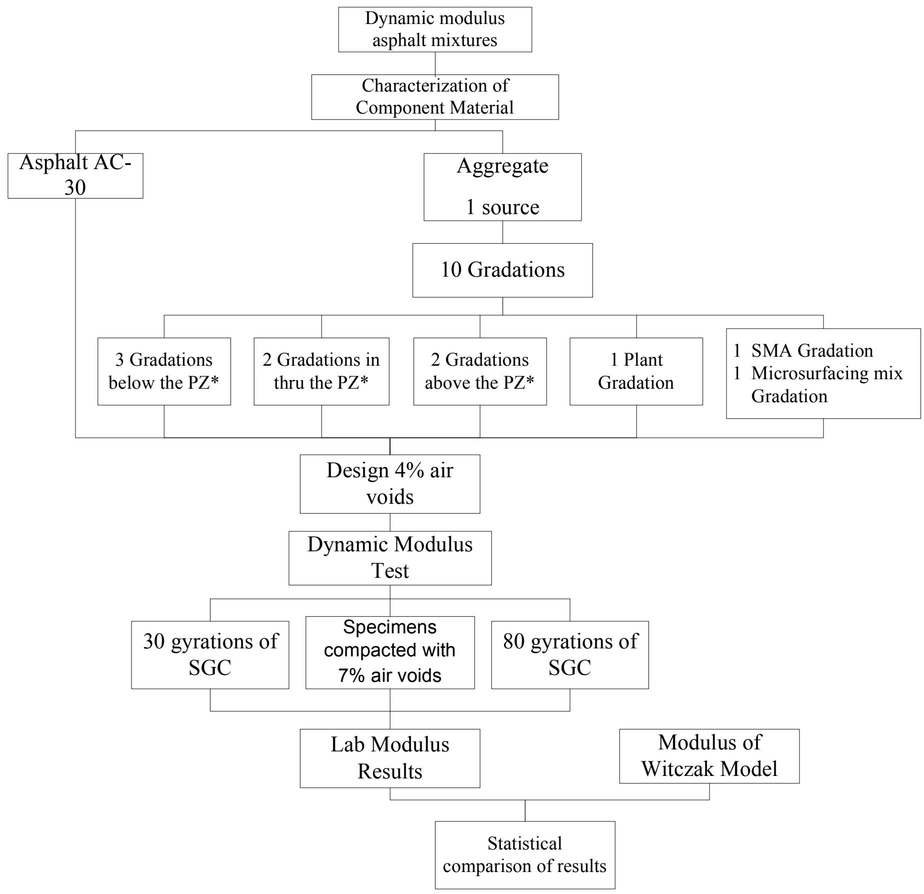
### *Asphalt Binder Properties*

In Costa Rica only one type of asphalt is produced. The binder viscosity classification corresponds to an unmodified AC-30. Based on the SUPERPAVE specification, the binder classifies as a PG64-22. The properties for the asphalt binder are shown in Table 2.

### *Specimen Preparation*

Ten different types of asphalt mixtures were designed in the laboratory. Three dense graded mixtures (G1, G2 and G3) below the "*prevention zone*" (also called SUPERPAVE's restricted zone); two dense graded mixtures (G6 and G7) above the "*prevention zone*"; two dense graded mixtures (G4 and G5) thru the "*prevention zone*"; one Stone Matrix Asphalt (SMA) mixture (G9); one micro surfacing mixture (G8), and a typical plant dense graded mixture (G10). The gradations are presented in Table 3 and Figure 2.

The design air void content was fixed to 4%. Two mixture design methodologies were used: Marshall and Superpave. The optimum asphalt content by dry weight of aggregate (DWA) and by total weight of mixture (TWM), voids in the mineral aggregate (VMA), the voids filled with asphalt (VFA), and the effective asphalt content (Pbe) based on both methodologies are shown in Table 4.




PZ*: "prevention zone" or SUPERPAVE's restricted zone

**FIGURE 1 Flow Chart for the Experimental Plan**

**TABLE 1 Physical Properties of the Aggregates Used in the Study.**

| Property | Test Method | Value | Unit | Specifications |
|---|---|---|---|---|
| **Coarse Aggregate** | | | | |
| L.A. Abrasion | AASHTO T 96 | 21.21 | % | 37% max.[1] |
| Specific Gravity | AASHTO T 85 | 2.652 | | 2.85 max.[1] |
| Absorption | AASHTO T 85 | 1.69 | % | 4% max.[1] |
| Faces Fractured 1 face | ASTM D 5821 | 100 | % | 90% min.[2] |
| 2 or more | | 99.8 | % | 75% min.[2] |
| **Fine Aggregate** | | | | |
| Plasticity index | AASHTO T 90 | NP | | 10% max.[1] |
| Sand equivalent | AASHTO T 176 | 78 | | - |
| Angularity | AASHTO TP 304 | 37.2 | % | - |
| Specific Gravity | AASHTO T 84 | 2.549 | | 2.85% max.[1] |
| Absorption | AASHTO T 84 | 3.283 | % | - |

[1] Nevada DOT Standard Specifications for Road and Bridge Construction, 2001.
[2] Standard Specifications for Constructions of Roads and Bridges on Federal Highways Projects, FP-03

**TABLE 2 Physical Properties of the Used Asphalt Binder.**

| Aging State | Property | Unit | Asphalt Binder AC-30 |
|---|---|---|---|
| Original | Density at 25ºC | $g/cm^3$ | 1.030 |
| | Absolute viscosity at 60ºC | Poise | 3330 |
| | Kinematic viscosity at 125ºC | centiPoise | 961 |
| | Kinematic viscosity at 135ºC | centiPoise | 565 |
| | Kinematic viscosity at 145ºC | centiPoise | 347 |
| | VTS, regression slope of viscosity temperature susceptibility | - | 3.43 |
| | Regression intercept | - | 10.26 |
| RTFO | Absolute viscosity at 60ºC | Poise | 11512 |
| | Kinematic viscosity at 125ºC | centiPoise | 1712 |
| | Kinematic viscosity at 135ºC | centiPoise | 938 |
| | Kinematic viscosity at 145ºC | centiPoise | 550 |

**TABLE 3 Studied Aggregate Gradations.**

| ASTM Sieve | Sieve (mm) | Studied Gradation | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | | Below the prevention zone | | | Thru the prevention zone | | Above the prevention zone | | Micro (*) | SMA | Plant |
| | | G1 | G2 | G3 | G4 | G5 | G6 | G7 | G8 | G9 | G10 |
| 3/4 | 19.0 | 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 |
| 1/2 | 12.5 | 95 | 100 | 90 | 95 | 95 | 98 | 90 | 100 | 90 | 95 |
| 3/8 | 9.5 | 88 | 95 | 78 | 90 | 90 | 92 | 65 | 81 | 45 | 79 |
| N°4 | 4.75 | 37 | 62 | 40 | 45 | 70 | 67 | 45 | 32 | 28 | 48 |
| N°8 | 2.36 | 28 | 33 | 32 | 37 | 50 | 47 | 42 | 27 | 23 | 32 |
| N°16 | 1.18 | 20 | 23 | 20 | 29 | 27 | 32 | 37 | 22 | 22 | 22 |
| N°30 | 0.60 | 13 | 16 | 14 | 22 | 15 | 23 | 30 | 18 | 19 | 16 |
| N°50 | 0.30 | 9 | 12 | 9 | 14 | 8 | 17 | 20 | 14 | 16 | 12 |
| N°100 | 0.15 | 7 | 9 | 7 | 9 | 6 | 12 | 12 | 10 | 13 | 8 |
| N°200 | 0.075 | 5 | 7 | 6 | 6 | 5 | 8 | 5 | 8 | 10 | 5 |

(*) Microsurfacing.

**TABLE 4 Summary Volumetric Properties of the Mix for All the Aggregate Gradations Studied.**

| Description | Gradation | Mix design | Va | Pb (DWA) | Pb (TWM) | Pbe | VMA | VFA |
|---|---|---|---|---|---|---|---|---|
| **Below the prevention zone** | **G1** | Superpave | 4.0% | 7.20 | 6.80 | 5.69 | 17.32 | 77.66 |
| | | Marshall | 4.0% | 6.41 | 6.02 | 5.18 | 15.74 | 74.66 |
| | **G2** | Superpave | 4.0% | 7.40 | 6.90 | 6.06 | 17.44 | 76.12 |
| | | Marshall | 4.0% | 6.84 | 6.40 | 5.49 | 16.51 | 75.78 |
| | **G3** | Superpave | 4.0% | 6.40 | 6.00 | 5.25 | 15.68 | 73.40 |
| | | Marshall | 4.0% | 6.01 | 5.67 | 4.83 | 15.15 | 71.93 |
| **Thru the prevention zone** | **G4** | Superpave | 4.0% | 5.50 | 5.30 | 4.31 | 12.14 | 73.20 |
| | | Marshall | 4.0% | 5.44 | 5.16 | 4.17 | 13.90 | 69.53 |
| | **G5** | Superpave | 8.0% | 7.50 | 7.00 | 6.00 | 20.90 | 61.60 |
| | | Marshall | 8.8% | 6.50 | 6.10 | 5.12 | 20.08 | 55.50 |
| **Above the prevention zone** | **G6** | Superpave | 4.0% | 5.50 | 5.20 | 4.35 | 14.10 | 72.10 |
| | | Marshall | 4.0% | 5.84 | 5.52 | 4.41 | 14.52 | 70.50 |
| | **G7** | Superpave | 4.0% | 5.00 | 4.80 | 3.32 | 12.32 | 63.20 |
| | | Marshall | 4.0% | 5.50 | 5.21 | 4.13 | 13.74 | 70.50 |
| **Micro surfacing** | **G8** | Superpave | 4.0% | 5.60 | 5.30 | 4.29 | 14.06 | 78.68 |
| | | Marshall | 4.0% | 5.99 | 5.65 | 4.51 | 14.82 | 71.00 |
| **SMA** | **G9** | Superpave | 4.0% | 4.90 | 4.70 | 3.74 | 12.44 | 68.86 |
| | | Marshall | 4.0% | 5.19 | 4.93 | 4.01 | 13.34 | 71.00 |
| **Plant** | **G10** | Superpave | 4.0% | 6.00 | 5.70 | 4.76 | 15.00 | 73.00 |
| | | Marshall | 4.0% | 5.65 | 5.35 | 4.46 | 14.50 | 71.10 |

***Dynamic Modulus of Asphalt Mixtures***

In order to evaluate the dynamic modulus of the different mixes, all specimens were prepared following the standard method ASTM D3496 *"Practice for Preparation of Bituminous Specimens for Dynamic Modulus Testing" (ASTM, 2005).* The testing was performed according to ASTM D3497 *"Standard Test Method for Dynamic Modulus of Asphalt Mixtures" (ASTM, 2007)* and AASHTO T 62 *"Determining Dynamic Modulus of Hot Mix Asphalt" (ASTM, 2007).*

The experimental design included four factors; the first factor was the gradation with the ten levels (G1, G2, G3, G4, G5, G6, G7, G8, G9 and G10), the second factor was the temperature with five levels (-5, 5, 20, 40 and 55°C), the third factor was the loading frequency with six levels (0.1, 0.5, 1, 5, 10 and 25 Hz), and the fourth factor was the compaction effort with three levels (30 gyrations of the Superpave gyratory compactor (SGC), 80 gyrations of SGC, and specimens compacted with 7% air voids).

***Master curves***

The master curves and the corresponding shift factors were developed directly from the dynamic modulus tests. Microsoft Excel Solver was used to optimize the calibration coefficients. It involved nonlinear optimization using the sigmoidal function shown in Equations 1 and 2. Both equations describe the time dependency of the modulus (The results are presented in Table 5 and Figure 2):

$$Log|E^*| = \delta + \frac{\alpha}{1 + e^{\beta + \gamma(\log t_r)}} \quad [1]$$

where,

$E^*$ = dynamic modulus.

$t_r$ = time of loading at the reference temperature.

$\delta$, $\alpha$ = estimated parameters; for a given set of data, $\delta$ represents the minimum value of E* and $\delta$+$\alpha$ represents the maximum value of E*.

$\beta$, $\gamma$ = parameters describing the shape of the sigmoidal function.

$$a(T) = \frac{t}{t_r} \quad , \quad \log(t_r) = \log(t) - \log[a(T)] \qquad [2]$$

where,

$t_r$ = time of loading at the reference temperature.

$t$ = time of loading at a given temperature of interest.

$a(T)$ = Shift factor as a function of temperature.

$T$ = temperature of interest.

**TABLE 5 Summary of the Fitting Parameters for the Construction of the E* Master Curves**

| Gradation | Parameter | | | |
|---|---|---|---|---|
| | $\delta$ | $\alpha$ | $\beta$ | $\gamma$ |
| **G1** | 1.8155 | 2.3618 | -0.5631 | 0.4766 |
| **G2** | 1.8647 | 2.4533 | -0.3800 | 0.5018 |
| **G3** | 1.8542 | 2.3952 | -0.3458 | 0.4784 |
| **G4** | 1.8013 | 2.5136 | -0.7055 | 0.4589 |
| **G5** | 2.1775 | 1.8860 | -0.1475 | 0.5982 |
| **G6** | 1.7743 | 2.7039 | -0.5207 | 0.4182 |
| **G7** | 2.1687 | 2.3301 | -0.5388 | 0.4960 |
| **G8** | 2.0420 | 2.0748 | -0.6264 | 0.5309 |
| **G9** | 2.0682 | 2.3802 | -0.6617 | 0.5529 |
| **G10** | 1.5471 | 2.7260 | -0.7342 | 0.4276 |

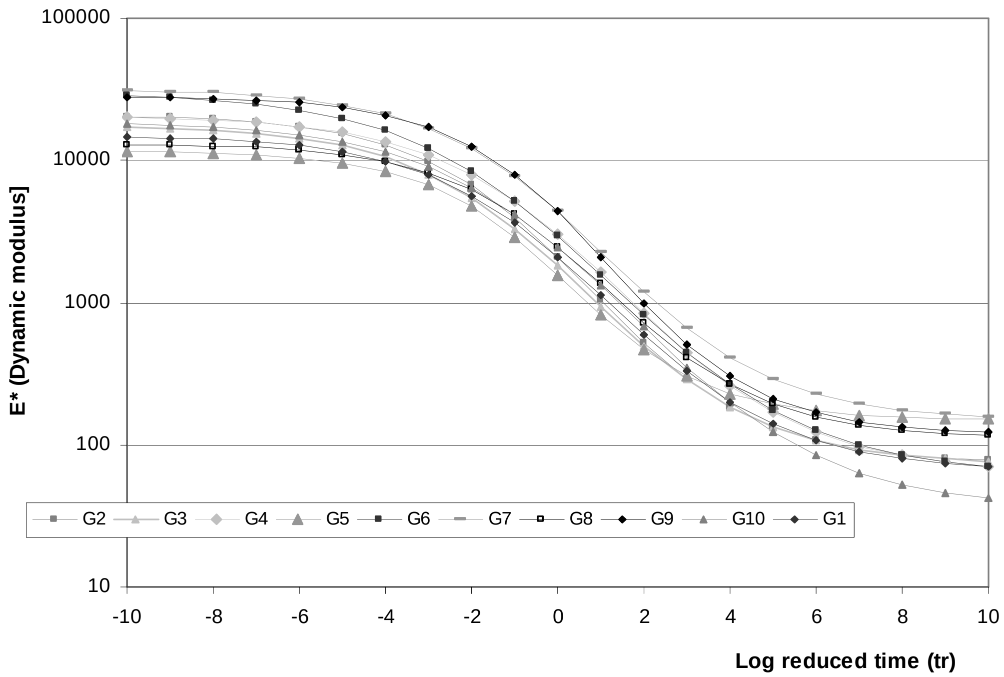



**FIGURE 2 Master Curves of Dynamic Modulus for the Gradations Used in the Study**

**Quality of fitted Witczak model on mixes in Costa Rica**

For Level 2 and Level 3 analysis, the master curves would be developed directly from the dynamic modulus Witczak I-37A predictive equation shown in equation 3. This equation is intended to predict the dynamic modulus of asphalt mixtures over a wide range of temperatures, rates of loading, and aging conditions based on information that is readily available from material specifications or volumetric design of the mixture (ARA, 2004).

$$logE^{*} = 3{,}750063 + 0{,}02932\rho_{200} - 0{,}001767(\rho_{200})^{2} - 0{,}002841\rho_{4} - 0{,}058097V_{a}$$

$$-0{,}802208\left(\frac{V_{beff}}{V_{beff}+V_{a}}\right) + \frac{3{,}871977-0{,}0021\rho_{4}+0{,}003958\rho_{38}-0{,}000017(\rho_{38})^{2}+0{,}005470\rho_{34}}{1+e^{(-0{,}603313-0{,}31335\log(f)-0{,}393532\log(\eta))}} \qquad [3]$$

where:
*E** = dynamic modulus, psi,
η = bitumen viscosity, 106 Poise,
*f* = loading frequency, Hz,
*Va* = air void content, %,
*Vbeff* = effective bitumen content, % by volume,
*ρ34* = cumulative % retained on the ¾ in sieve,
*ρ38* = cumulative % retained on the 3/8 in sieve,
*ρ4* = cumulative % retained on the No. 4 sieve,
*ρ200* = % passing the No. 200 sieve.

The overestimation of E* values for mixtures in Costa Rica (Figure 4) was reported by Loría and his associates (Loría et al., 2011). The application of this model on mixtures in Costa Rica not only over predicted E* values but also failed to comply one of the assumptions of OLS regression and ANOVA: a constant variance of the error term. In the residual versus the fitted values plot, the errors should have constant variance when the residuals are scattered randomly around zero. In this case, the residuals increase or decrease with the fitted values in a pattern that looks like a funnel or uneven spreading of residuals across fitted values, the errors may not have constant variance (Figure 3). A curvilinear pattern in the residual versus fitted values plot also indicated that a higher-order term to has to be added model.

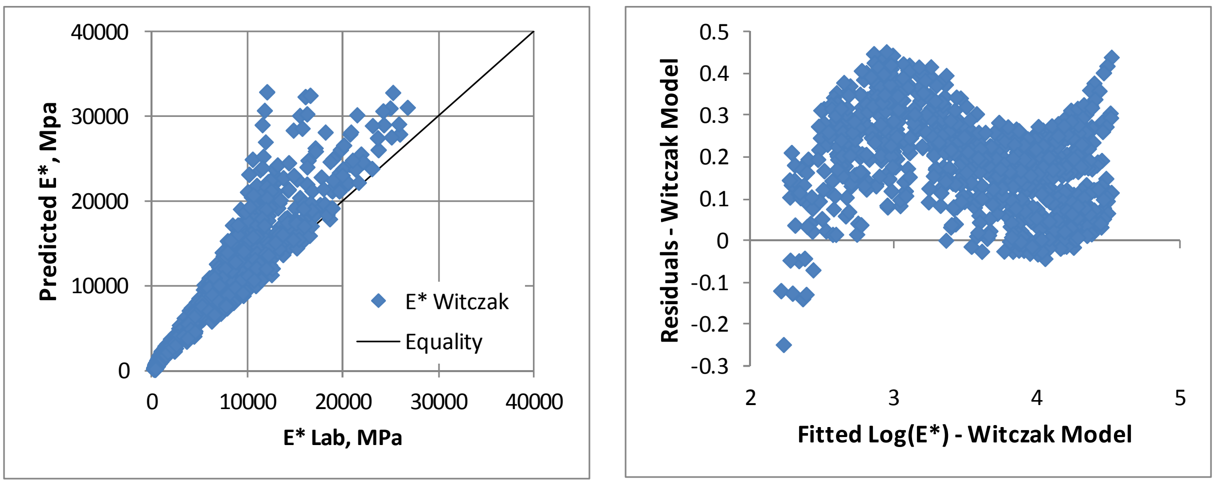



**FIGURE 3 Evaluation of the Witczak Model**

As mentioned previously, the Witczak-Lanamme model was developed to adjust or calibrate the Witczak model based on the E* results of several mixtures used in Costa Rica. This new model was suggested based on a nonlinear approach that significantly improved the model fit ($R^2$=0.9355, standard deviation of error term=1,494.4 and SSE=5.1997). The model is shown in Equation 4.

$$\log E^* = 5{,}535833 + 0{,}002087\rho_{200} - 0{,}000566(\rho_{200})^2 - 0{,}002590\rho_4 - 0{,}078763V_a$$
$$-1{,}865947\left(\frac{V_{beff}}{V_{beff}+V_a}\right) + \frac{2{,}399557 + 0{,}000820\rho_4 - 0{,}013420\rho_{38} + 0{,}000261(\rho_{38})^2 + 0{,}005470\rho_{34}}{1+e^{(0{,}052941-0{,}498163\log(f)-0{,}691856\log(\eta))}} \qquad [4]$$

The application of the Witczak-Lanamme model not only fixed the overestimation of E* values but also in the residual versus the fitted values plot, the errors had constant variance with the residuals scattered randomly around zero (Figure 4). However, further investigation showed that high errors were still obtained from some mixtures and the variance of the predictions was not uniform. Ideally, in the plot of actual E* values versus predicted ones, a small and random deviation from the line of equality is desired for all data points. As an attempt to reach this ideal scenario, the artificial neural networks (ANN) methodology was implemented using the same dataset.

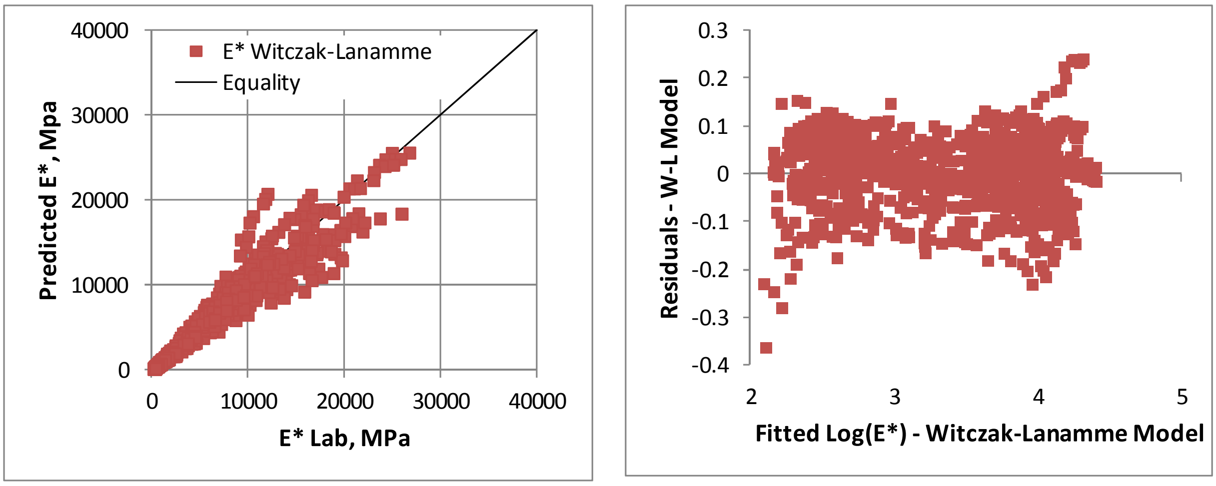



**FIGURE 4 Evaluation of the Witczak-Lanamme Model**

**Development of the ANN-Lanamme Model**

An artificial neural network is a massively parallel distributed processor that has a natural propensity for storing experimental knowledge and making it available for use (*Priddy and Keller, 2005*). Consequently, knowledge is acquired by the network through a learning (training) process; the strength of the interconnections between neurons is implemented by means of the synaptic weights used to store the knowledge. The learning process is a procedure of adapting the weights with a learning algorithm in order to capture the knowledge. In other words, the aim of the learning process is to map a given relation between inputs and outputs of the network.

The learning method used to develop the ANN models was a feed-forward back propagation with the sigmoidal function, Equation 5, as the transfer function. It was found that the two-layer network with 10 nodes in the hidden layer was the most appropriate for this dataset (Figure 5). The basic form of the ANN is given by Equations 5 through 7. For these equations, a single index indicates an array; dual indices represent a matrix with the first letter indicating the values in the row and the second letter indicating the values in the column. The index *i* represents the input parameters and the index *k* represents the hidden layer.

$$f(T) = \frac{2}{1+e^{-2T}} - 1 \qquad [5]$$

$$H_k^1 = B_k^1 + \sum_{i=1}^{m} W_{ik}\ P_i \qquad [6]$$

$$Output = Ln(E^*) = f\left(B_O + \sum_{j=1}^{m} H_k^1 W_k\ \right) \qquad [7]$$

where;
$T$ = placeholder variable,
$H_k^1$ = transferred value of nodes at the hidden layer,
$P_i$ = input variables (ρ200, ρ4, ρ38, Va , $V_{beff}$ , loglog(η), temperature and frequency),
$W_{ik}$ = weight factors for the hidden layer,
$W_k$ = weight factors for the output layer,
$B_k^1$ = bias factors for first layer,
*B*0 = bias factor for outer layer,
*m* = number of nodes in hidden layer
*Ln(E*)* = natural logarithm of E*.

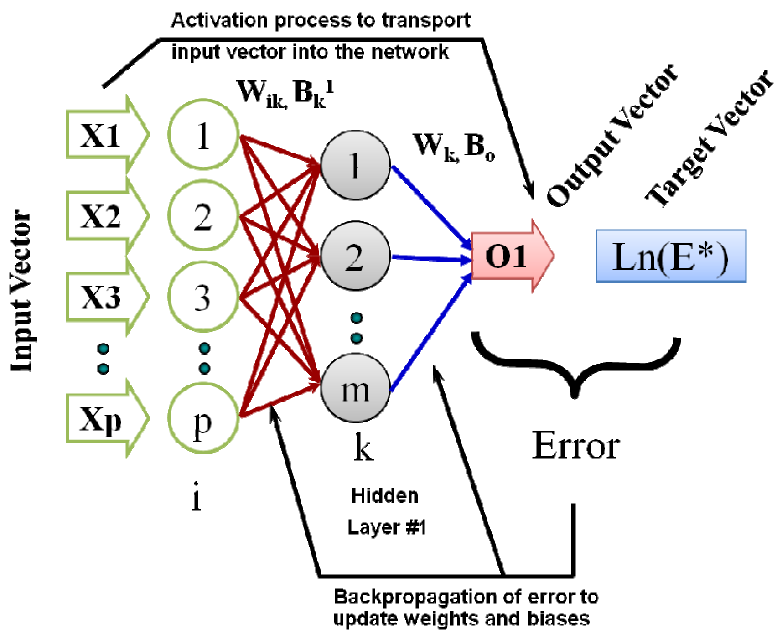



**FIGURE 5 Schematic of training process.**

***Model weights and bias values***

The following shows the weight matrices and bias vectors for the ANN-Lanamme model. If these parameters are substituted into Equations 5 through 7 with ρ200 = 5, ρ4 = 63, ρ38 = 12, Va = 7.41, $V_{beff}$ =

5.69, loglog(η) = 3.7859, temperature = -3.4 and frequency = 25, the predicted ln($E$*) value of 9.31415 and the E* value of 11,402 MPa would be calculated from this recommended model.

$$B_k^1 = [2.3134 \quad 4.0247 \quad 2.1380 \quad -11.9793 \quad 0.3330 \quad -6.3721 \quad -5.0298 \quad -0.2873 \quad -10.6756 \quad 10.3805]$$

$$W_{ik} = \begin{vmatrix} -4.2794 & 23.2425 & -4.0547 & -12.9996 & 0.0060 & 8.4144 & -3.4470 & -0.0002 & 0.0118 & 13.7398 \\ -10.8394 & -2.4254 & -5.4623 & 4.0784 & -0.0272 & -2.1539 & -7.4460 & 0.0250 & 0.0073 & -17.2411 \\ 7.5808 & 5.2567 & 7.6583 & -22.1995 & 0.1250 & -2.7495 & 0.8265 & -0.1192 & -0.0303 & -5.5932 \\ -15.3861 & 26.5062 & -1.7360 & 3.2115 & 0.1505 & 0.9341 & -18.8182 & -0.1521 & -0.0390 & -3.5607 \\ 2.3739 & -5.2556 & 8.2966 & -4.0598 & 0.1879 & 3.7634 & 3.6738 & -0.1812 & -0.0185 & 5.0671 \\ -0.3161 & -6.6774 & -4.7862 & 0.2819 & 0.4871 & 1.6285 & -0.4955 & -0.4942 & -1.8913 & 4.2234 \\ -0.1810 & -14.1131 & -8.9340 & 2.1166 & 1.3467 & 2.3166 & -0.5999 & -1.3313 & -1.8068 & 2.0520 \\ 0.0159 & 0.6328 & 0.4785 & -0.1558 & -0.3746 & -0.1602 & 0.0106 & 0.3321 & -8.0241 & 0.1222 \end{vmatrix}$$

$$W_k = \begin{vmatrix} 0.2402 \\ 0.0377 \\ 0.0410 \\ 0.0722 \\ 6.6041 \\ -0.0695 \\ -0.2727 \\ 7.6429 \\ -13.6531 \\ 0.0456 \end{vmatrix} \qquad B_0 = -13.6918$$

The results of the application of the ANN-Lanamme model on the entire E* database are shown in Figure 6. In the plot of actual E* values versus predicted ones, a small-constant deviation from the line of equality was acquired for all data points. In addition, in the residual versus the fitted values plot, the errors had constant variance with the residuals scattered randomly around zero.

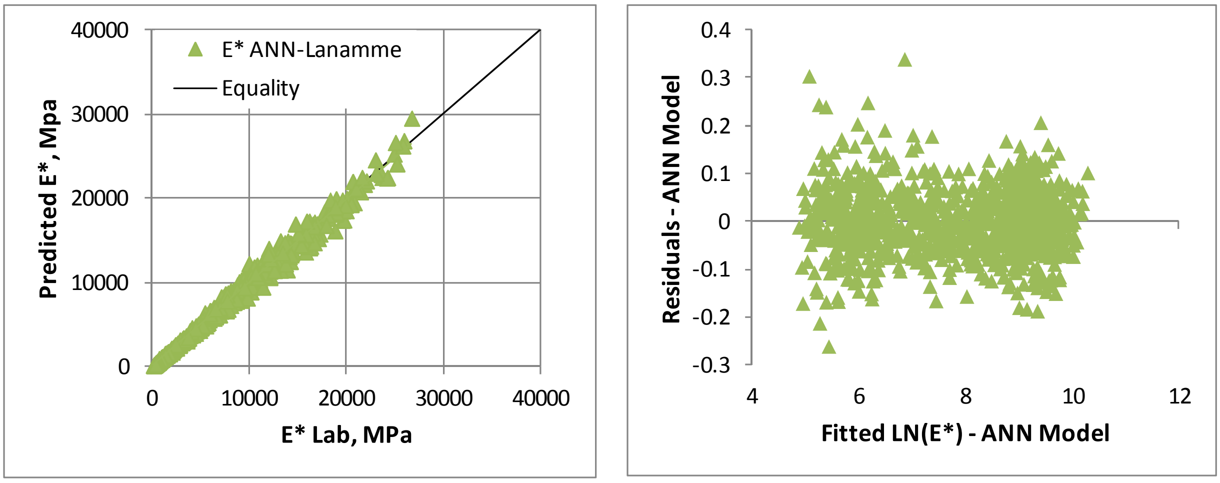



**FIGURE 6 Evaluation of the ANN-Lanamme Model.**

The slope of the measured versus predicted curve for all three models was used to perform a bias analysis. As shown in Figure 7 the highest deviation from the line of equality in terms of the slope was obtained for the Witczak I-37A model. On average, predicted E* values were deviated from the equality line by 35%. In second place, the Witczak-Lanamme model predicted E* values with a slight deviation from the equality line (about 4%). Finally, the ANN had the lowest deviation from the line of equality with only 1%.

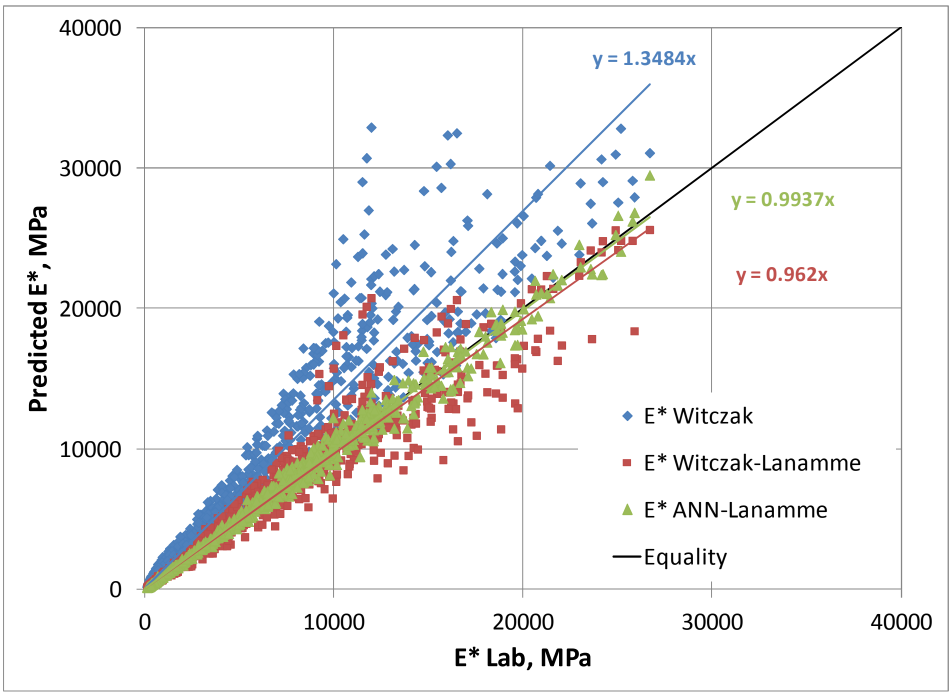



**FIGURE 7 Comparison of Predictive Models.**

The accuracy of the three predictive models was also analyzed by means of goodness of fit parameters in arithmetic space. Table 6 shows the calculated parameters for all the models along with the the respective criteria. The lowest coefficient of determination ($R^2$) in arithmetic space and the highest standard error of the estimate/standard deviation value (Se/Sy) for the Witczak model confirmed its limited ability to predict E* values for mixtures in Costa Rica. A significant improvement in the prediction of E* values, with respect to the Witczak model was obtained with the use of the Witczak-Lanamme model (59% improvement in the R2 value and 29% reduction in the Se/Sy value). However, the best results were obtained for the ANN-Lanamme model with the highest $R^2$ and lowest Se/Sy values (69% improvement in the $R^2$ value and 77% reduction in the Se/Sy value with respect to the Witczak model).

**TABLE 6 Goodness of Fit Parameters**

| Parameters | | | | Goodness of Fit (Witczak et al. 2002) | | |
|---|---|---|---|---|---|---|
| **Model** | **$R^2$** | **$R^2$ adj.** | **Se/Sy arithmetic** | **Criteria** | **$R^2$** | **Se/Sy** |
| Witczak | 0.592 | 0.589 | 0.372 | Excellent | > 0.90 | < 0.35 |
| Witczak-Lanamme | 0.935 | 0.934 | 0.262 | Good | 0.70 - .089 | 0.36 - 0.55 |
| ANN-Lanamme | 0.993 | 0.992 | 0.086 | Fair | 0.40 - 0.69 | 0.56 - 0.75 |
| | | | | Poor | 0.20 - 0.39 | 0.76 - 0.90 |

In summary, overestimation of E* values for Costa Rican mixtures by the Witczak I-37A model led to its local calibration (Witczak-Lanamme model). An additional model adequacy checking performed on these two models led to the construction of a new and improved model based on artificial neural networks (ANN-Lanamme model). This final model not only met the model adequacy criteria but also had the best overall goodness of fit parameters.

## CONCLUSIONS AND RECOMMENDATIONS

Even when a local calibration of the Witczak I-37A model was performed for 10 mixtures in Costa Rica, there was still room for improvement. Further investigation showed that high errors were still obtained from some mixtures when using the calibrated model. Additionally, the data clearly indicated that calibration of the E* models, based on direct application of standard regression techniques such as OLS was not adequate since several of the assumptions made when using this technique were violated, rendering the estimated values as inefficient (variance in the model can be further improved).

The application of artificial neural networks proved to be a most appropriate methodology to improve the predictability of E* values. The ANN-Lanamme model complied with the model adequacy criteria, had the best goodness of fit parameters and exhibited the lowest overall bias (69% improvement in the $R^2$ value and 77% reduction in the Se/Sy value with respect to the Witczak model).

In order to further improve this prediction model for Costa Rica, future calibration and verification efforts are necessary; therefore, it is recommended to increase the number of tests performed (increase the E* database).

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